

Multifractal Analysis of Solar Magnetograms

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Abstract. As solar observational techniques improve, the fine small-scale structure observed on the solar surface becomes more pronounced. Complex filigree structures of solar granulation, sunspots, photospheric magnetic and velocity fields cannot be described adequately by a single parameter (e.g., filling factor, fractal dimension, or power law index). Methods which incorporate parameters that are a function of scale (multi-scale methods) to describe the complexity of a field under study, should be involved. The multifractal approach offers such a possibility. In this paper the scaling of structure functions is proposed in order to analyze multifractality. Application of the approach to SOHO/ MDI high resolution magnetograms of active regions show that the structure functions differ for all active regions studied. For a given active region, the functions may maintain their shape during several hours, however they can significantly change during a day. Flare-quiet active regions tend to possess a lower degree of multifractality than flaring active regions do. The increase in multifractality is a signal that a magnetic structure is driven to a critical state, thus gaining tangential discontinuities of various length scales.

Keywords: Sun: fractals, solar magnetic fields

1. Introduction

Luxuriant and filigree character of structures observed on the Sun impels us to apply fractal methods (Mandelbrott 1977, Feder 1988, Takayasu 1989) in order to learn more about these structures, in particular, about magnetic fields (Balke et al. 1993, Meunier 1999, Lawrence et al. 1993, McAteer et al. 2005).

A fractal is an object whose parts somehow resemble the whole, or, to put it in more rigorous way, a self-similar, scale-invariant structure without any characteristic length (Feder 1988, Takayasu 1989).

In this paper I concentrate on two-dimensional fractals. Several approaches were elaborated to explore the fractal nature of 2D geo-

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metrical objects. The approaches differ by mathematical measures introduced on the geometrical object (Takayasu 1989, Lawrence et al. 1993). The method proposed in the present paper (Section 2) is based on a measure defined by structure functions. However, in order to illustrate the idea of multifractality in Introduction, I will use the most popular approach based on a probability measure, which leads to a notion of a fractal dimension. Let us the fractal to be covered by a 2D mesh of square cells of side r , which is varied. Number of boxes, N , occupied by the fractal is proportional to the scale length, r , in a fractional power, D_f : $N(r) \propto r^{D_f}$, which is called a fractal dimension. The fractal dimension can be approximated as a slope of $N(r)$ versus the inverse scale, $1/r$, in a double-logarithmic plot (Figure 1). Linearity of $\log(N(r))$ versus $\log(1/r)$ is called self-similarity. Computer-generated fractals are self-similar over an infinite range of scales. Such structures can be represented by one scalar parameter, D_f , and they are usually called monofractals.

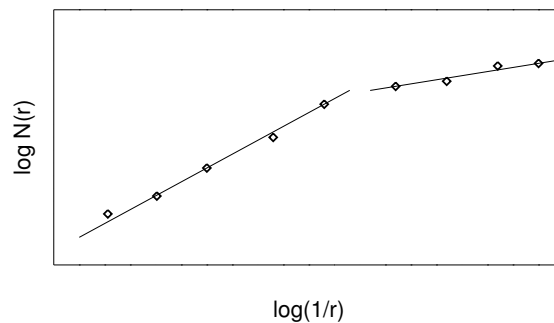


Figure 1. A slope of $\log(N)$ versus $\log(1/r)$ defines the fractal dimension, D_f . For natural fractals (i.e. for multifractals), the slope can change for different scale ranges.

However, in the case of natural fractals, self-similarity does not hold in the strict sense but does hold statistically over a finite range of scales. As soon as we deal with natural fractals, we immediately reveal that the slope of $\log(N)$ versus $\log(1/r)$ can change for different scale ranges (Figure 1). The cause of this may be as the nature of fractals so unavoidable influence of the process of data acquisition and processing. However, the later is out of the scope of this paper (see, e.g., Lawrence et al 1996 for more details on the

estimation of the fractal dimension from digitized images of solar magnetic fields). Now, the entire range of scales can be divided into an infinite number of subsets, and for each subset its own self-similarity law holds with its own fractal dimension. Thus, a natural fractal may be treated as a multifractal, i.e. a superposition of an infinite number of monofractals. Local (in sense of scale) character of self-similarity for natural fractals is one of the reasons to make a step from monofractals to multifractals.

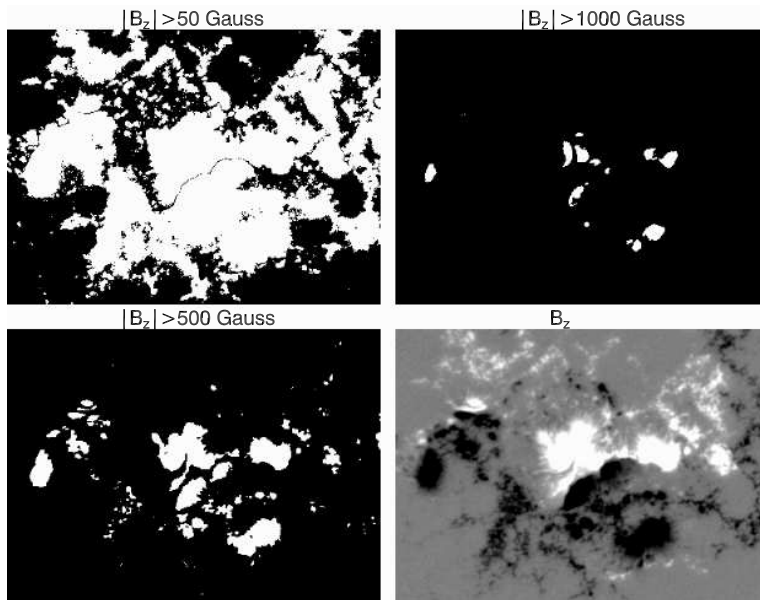


Figure 2. An hierarchy of binary masks of a solar magnetic structure (lower right frame). Each binary mask (left frames and upper right frame) is equal to unity on the set of all pixels where the absolute value of the magnetic field exceeds a given threshold (noted above the frames) and zero elsewhere. Superposition of infinite number of such masks/monofractals results in the observed magnetic structure, which is a multifractal (see the text).

Another reason is an our intention to take into account the distribution of density in a fractal. In the case of solar magnetic fields, when we define a magnetic area as a binary mask (Figure 2) equal to unity on the set of all pixels where the absolute value of the magnetic field exceeds a given threshold (for example, 50 G, Figure 2, upper left frame), and to zero elsewhere, we obtain a fractal. By using, for example, the box-counting method, or some other methods, (Takayasu 1989), we can calculate the fractal dimension of the binary mask, which characterizes complexity of its bound-

ary. However, the fractal dimension can tell us nothing about the distribution of the magnetic field density *inside* the mask. It is well known that the density distribution may not be homogeneous. Moreover, the distribution of density determines the complexity of the magnetic structure and, therefore, the underlying physics. Thus, we have to incorporate the information on density. This can be done by introducing an hierarchy of masks defined by increasing threshold. The idea is illustrated in Figure 2: by specifying progressively higher thresholds, say, 500 G (lower left frame), 1000 G (upper right frame), we obtain new fractals and their parameters may vary. By incorporating these new monofractals, we can obtain addition information on the distribution of more and more dense patters of the magnetic structure. The superposition of the infinite number of the monofractals results in a multifractal: a real distribution of the solar magnetic field (see lower right frame in Figure 2. Please, note that for simplicity sake, the sign of B_{\parallel} on the masks was disregarded.) For a multifractal, a continuous spectrum of fractal parameters - a multifractal spectrum - is needed to characterize the structure. The broader the multifractal spectrum, the more complex the structure with higher degree of multifractality it represents.

One more reason to incorporate multifractality is the existence of a highly intermittent character of structures arising in the evolution of the dynamic systems, where dissipative processes may not be neglected (such processes produce a majority of natural fractals). Intermittency implies a tendency of a certain quantity to concentrate into small-scale features of high intensity surrounded by extended areas of much lower fluctuations (Monin & Yaglom 1975). Intermittency manifests itself via the burst-like behavior in temporal and spatial domains. Figure 3 demonstrates the phenomenological difference between a Gaussian process (which is non-ittemittent) and an intermittent process. Large fluctuations in the intermittent process are not so rare as in the Gaussian process, and they contribute significantly into the statistical moments which leads to multifractality (see Frisch 1995 for more details). Thus, by analyzing multifractality of a structure, we will also explore its intermittency. Generally, intermittency and multifractality are two different terms for the same phenomenon. Historically, the former term is usually applied to time series analysis, whereas the later one is used for spatial objects (Takayasu 1989, Frisch

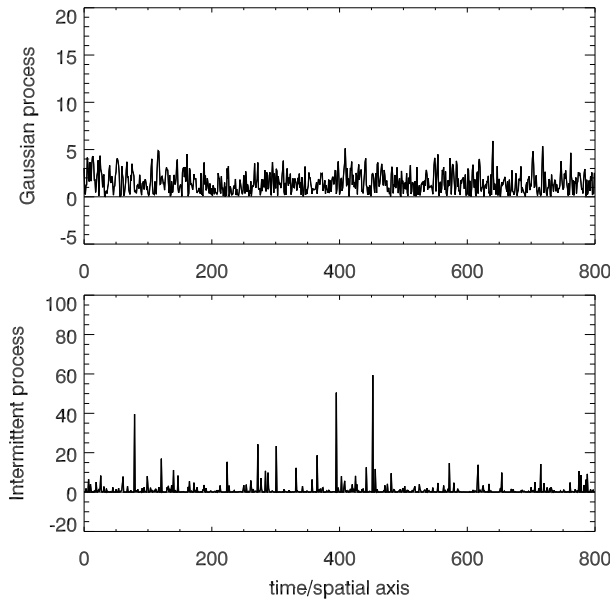


Figure 3. Illustration of phenomenological difference between a Gaussian process (upper frame), which is a non-intermittent and monofractal process of extremely rare large fluctuations, and an intermittent process (lower frame), which is a multifractal due to the contribution of large fluctuations (which are not so rare now) into the statistical moments. Horizontal axis may represent a time or a spatial coordinate.

1995). It is worthwhile to note that when a field is intermittent, the structure of its energy dissipation can not be homogeneous, but is intermittent, too (Monin & Yaglom 1975, Frisch 1995). Thus, by exploring intermittency of the magnetic field, one may also obtain valuable information on the behavior of the magnetic energy dissipation (Lawrence et al. 1995, Abramenko 2003, Vlahos & Georgoulis 2004).

There are several techniques to evaluate the degree of multifractality. In particular, for solar line-of-sight magnetograms, the box-counting method based on changing of the coarse-graining level was used by Lawrence et al (1993) and by Cadavid et al. (1994) to calculate the spectrum of fractal dimensions. Another methods are based on the fractal measure relations, calculations of correlation and distribution functions (Takayasu 1989), or structure functions (Frisch 1995). Here I will show how structure functions can be used to evaluate the degree of multifractality in order to obtain benefits

in solar image processing and learn more about solar magnetic fields.

2. Scaling of structure functions

Structure functions, defined as statistical moments of the increment of a field, are very useful non-traditional tool to study multifractality and intermittency (Stolovitzky & Sreenivasan 1993, Frisch 1995, Consolini et al. 1999, Abramenko et al. 2002).

In our case, the analyzed field is the line-of-sight component of the photospheric magnetic field, B_{\parallel} , so a structure function can be defined as follows:

$$S_q(r) = \langle |B_{\parallel}(\mathbf{x} + \mathbf{r}) - B_{\parallel}(\mathbf{x})|^q \rangle, \quad (1)$$

where \mathbf{x} is each point on a magnetogram, \mathbf{r} is the separation vector, and q is the order of a statistical moment, which takes on real values. Angular brackets denote averaging over the magnetogram. Within some range of scales, when plotted in a double logarithmic plot, structure functions are linear. The slope, $\zeta(q)$, is then a function of the statistical moment, q . Figure 4 shows an example of the structure functions (upper left frame) and the corresponding $\zeta(q)$ function (upper right frame) calculated from one line-of-sight magnetogram of an active region.

When the field under study is a monofractal, then $\zeta(q)$ is a straight line. However, for multifractals, $\zeta(q)$ appears to be a concave function and then its derivative $h(q) = d\zeta(q)/dq$ is not a constant value, so that h varies within some range Δh . For each value of h there is a fractal with an h -dependent dimension, $D(h)$. (According to Frisch (1995), h and $D(h)$ are related by the Legendre transform, however, calculation of $D(h)$ is beyond the scope of this paper). Thus, multifractality of the field is manifested via concavity of $\zeta(q)$ function. I will accept the value $\Delta h \equiv h_{max} - h_{min}$ (Figure 4, lower right frame) as an estimation of the degree of multifractality of the field. Indeed the larger Δh , the broader the set of monofractals which together form the resulting multifractal.

The weakest point in the above technique is determination of the scale range, Δr , where the slope $\zeta(q)$ is to be calculated. To visualize the range of multifractality, Δr , I suggest to use the flatness function. Generally, the ratio of the fourth statistical moment

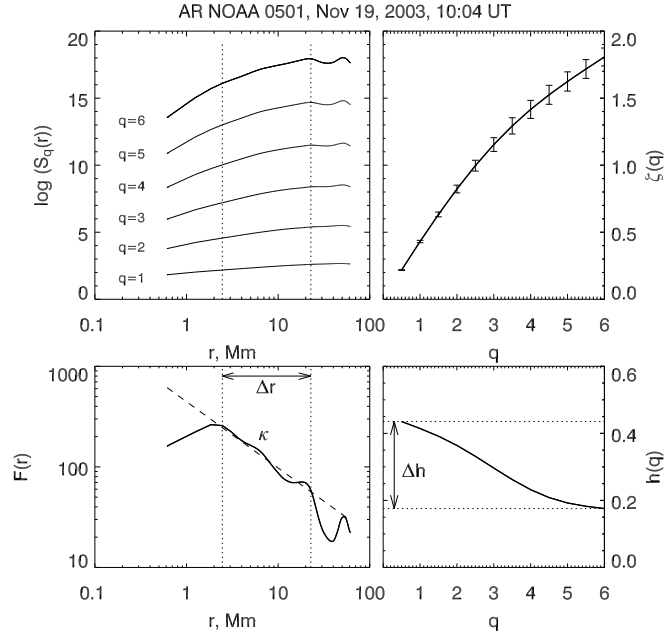


Figure 4. Structure functions $S_q(r)$ (*upper left*) calculated from a magnetogram of active region NOAA 0501 by Equation (1). *Lower left:* - flatness function $F(r)$ calculated from the structure functions by Equation (2). Vertical dotted lines mark the interval of multifractality, Δr , where flatness grows as power law when r decreases. The interval Δr is also marked in upper left frame. κ is the power index of $F(r)$ determined within Δr . The slope of $S_q(r)$, defined for each q within Δr , is $\zeta(q)$ function (*upper right*), a concave for a multifractal and straight line for a monofractal. *Lower right:* - function $h(q)$ is a derivative of $\zeta(q)$. The interval between the maximum and minimum values of $h(q)$ is defined as a degree of multifractality, Δh .

to the square of the second statistical moment determines the flatness function. However, when analyzing multifractality, Frisch (1995) suggested to use higher statistical moments to calculate the (hyper-)flatness, namely, the ratio of the sixth moment to the cube of the second:

$$F(r) = S_6(r)/(S_2(r))^3. \quad (2)$$

Equation (2) was used to calculate $F(r)$ and, for simplicity, I will refer to $F(r)$ as the flatness function. For monofractal structures, the flatness does not depend on the scale, r . On the contrary, for a multifractal structure, the flatness grows as a power-law, when the scale r decreases (Frisch 1995). Usually, when studying

natural fractals, the range Δr of the flatness's growth is bounded (see lower left frame in Figure 4). Possible causes of the boundness of the interval of multifractality were not comprehensively studied yet (Frisch 1995).

The slope, κ , of flatness function, determined within Δr , (Figure 4) seems to be related to the degree of multifractality: the steeper the function $F(r)$, the higher the degree of multifractality.

3. Diagnostics of image defects

As it was discussed above, a random field with Gaussian distribution of density is a monofractal. The method was applied first to such a Gaussian field with a zero mean value and the standard deviation $\sigma_0 = 230$ G, that was chosen to be the same as those obtained from high resolution magnetograms of active regions. The result is presented in Figure 5, and it shows that for a monofractal structure, the code produces the correct result with $\Delta h = 0$ and $F(r) = const.$

Then the standard deviation was increased three-fold, 5×5 point box-car running averaging was applied and the calculations were repeated again. In this case, a constant flatness and vanishing degree of multifractality were also obtained (figures are not shown). This experiment shows that results of the study of the multifractality, obtained by the proposed method, seem to be not contaminated by the changes in the noise level and seeing, which, plausibly, produce Gaussian structures.

However, when an artificial "hot" pixel was added (a pixel with extremely high artificial density) in the magnetogram, the flatness begins to curve up at small scales (Figure 6). The reason for that is that the presence of extremely large fluctuations implies intermittency, in other words, multifractal organization of a field (see Figure 3). This property of flatness was used to automatically select and discard magnetograms with "hot" pixels present in them.

Next, a large-scale distortion in a magnetogram was modeled in order to mimic an uncorrected flat field: to the smoothed Gaussian magnetogram a gradual function $A \cdot \cos(2\pi xL)$ was added. Here x is the horizontal coordinate on the map, L is the total size in the x direction and A is a free parameter (see Figure 7, lower right

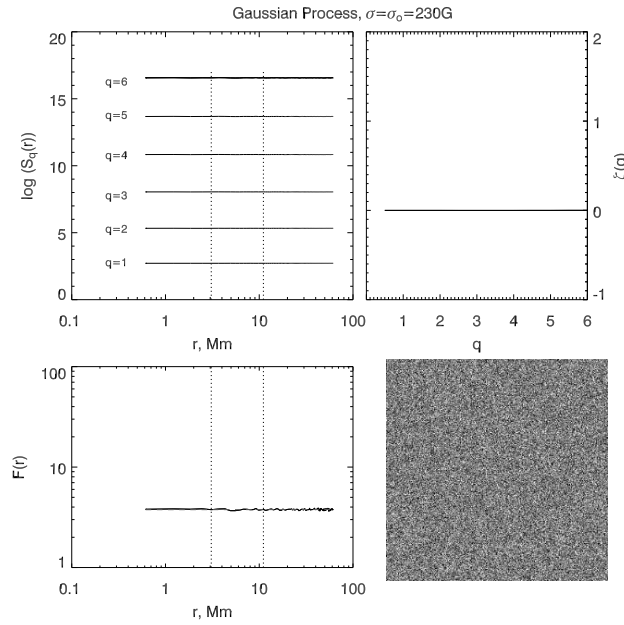


Figure 5. Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function calculated from an artificial Gaussian field (*lower right*). Other notations are the same as in Figure 4.

frame). For this “uncorrected” magnetogram, the flatness function, $F(r)$, curves down at large scales (see Figure 7, lower left frame). Therefore, this property of flatness may be used for diagnostics of the quality of the flat field correction.

An experience in analyzing a sequence of magnetograms of high time cadence (say, 1 minute) for a given active region showed that functions $S_q(r)$, $F(r)$, $\zeta(q)$ and $h(q)$ vary very gradually from one magnetogram to another. Any defect in a magnetogram caused by the instrument (e.g., “hot” pixels, vignetting, flat field non-uniformity, etc.) is immediately reflected via sharp changes in the shape of the flatness function and, to a lesser degree, in the shape of the rest of the functions. This property of $F(r)$ was used to select and discard automatically bad quality magnetograms.

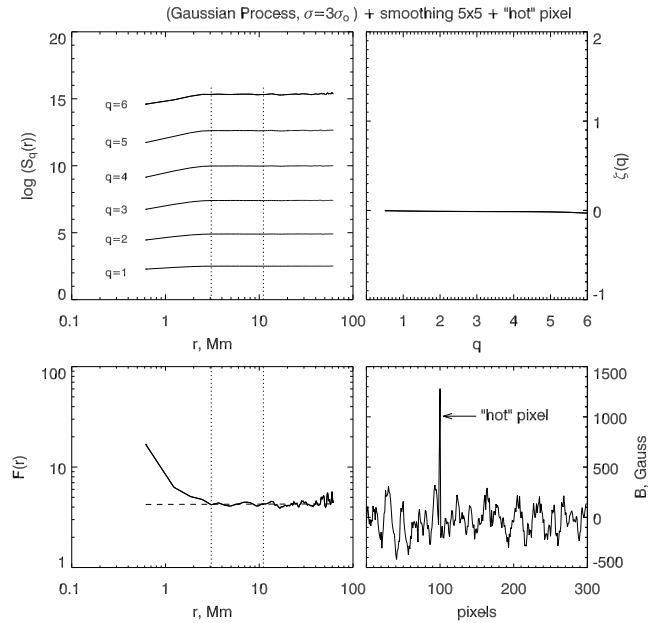


Figure 6. Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function, calculated from a smoothed Gaussian field with artificially added extremely high fluctuation in one of pixels (“hot” pixel). *Lower right*: a scan across the “hot” pixel. Other notations are the same as in Figure 4.

4. Multifractality of the magnetic field in active regions

As for any other multiscale method, data of highest available resolution and sensitivity are desirable to use for the proposed code in order to trace finest real features. Moreover, different active regions observed with *the same* instrument should be analyzed at the first stage of the code testing. Thus, in the present study the code was applied to magnetograms of different solar active regions obtained in the high resolution (HR) mode by the Michelson Doppler Imager (MDI) instrument (Scherrer et al., 1995) onboard the Solar and Heliospheric Observatory (SOHO). The pixel size of the magnetograms is 0.58×0.58 arcsec. The HR mode of SOHO/MDI can register active regions located only within the central parts of the solar disk.

The next stage of the code testing will be the comparison of results obtained with different instruments, which is a subject for

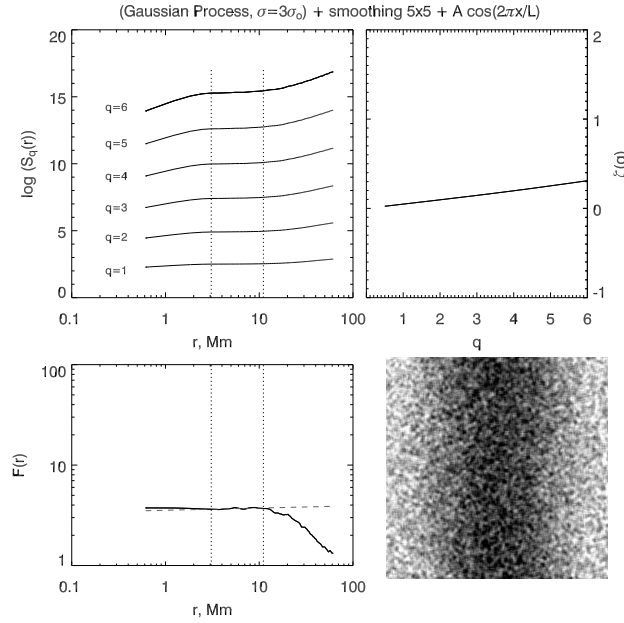


Figure 7. Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function, calculated from an artificial magnetogram (*lower right*) which is a smoothed Gaussian field with artificially added function $A \cdot \cos(2\pi x/L)$. Other notations are the same as in Figure 4.

further publications. Application of the first version of our code (without the flatness function) to the ground-based data can be found in Abramenko et al. (2002, 2003).

The result of calculations for two active regions, NOAA 9077 and 0061, are presented in Figures 8 and 9, respectively. One can see that scaling behavior of the structure functions is rather different in these active regions. For the first one, NOAA AR 9077, there exists a well-defined range of scales, $\Delta r = (4-23)$ Mm where flatness $F(r)$ grows with the power index $\kappa = -1.17$ as r decreases. Function $\zeta(q)$ is concave and the corresponding $\Delta h \approx 0.5$. The result implies a multifractal structure of the magnetic field in this active region.

On the contrary, in NOAA AR 0061 (Figure 9) the flatness function oscillates around a horizontal line, which implies a monofractal character of the magnetic field. The function $\zeta(q)$ is almost a straight line that produces a vanishing value of $\Delta h \approx 0.05$.

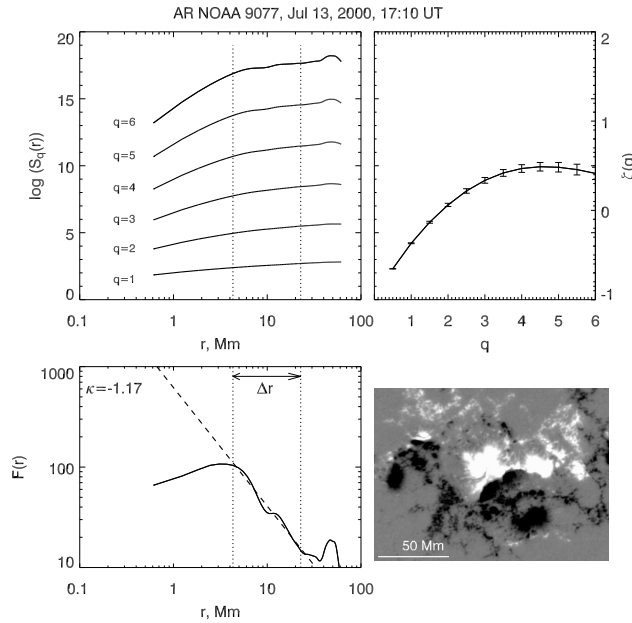


Figure 8. Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function from one magnetogram of active region NOAA 9077 (lower right). κ is a power law index of $F(r)$ calculated inside Δr . The degree of multifractality for this magnetogram $\Delta h = 0.48$. Other notations are the same as in Figure 4.

Time profiles of Δh for the two active regions are compared in Figure 10. Two six-hour data sets allowed us to conclude that NOAA AR 0061 persistently displays lower degree of multifractality, as well as lower level of X-ray emission than NOAA AR 9077 does. Note that NOAA AR 0061 was the only strong feature on the solar disk, thus the GOES flux time profile can be associated with the activity in this region. As to NOAA AR 9077, Big Bear Solar Observatory Ca II K images show that the gradual M1.2 flare (marked by an arrow in Figure 10) was launched in a mature active region 9077, whereas other GOES-flux peaks during the time period under study seem to be associated with another, rather moderate, active regions on the periphery of the solar disk.

It is appropriate to mention here that the two active regions displayed very different flare productivity. Active region 9077 was very flare productive: during the passage across the solar disk, it launched 37 flares of X-ray class higher than C3, including thirteen

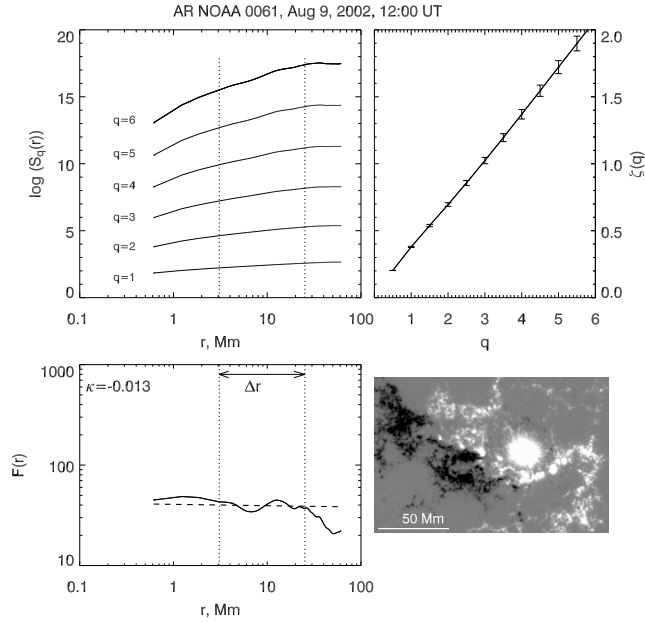


Figure 9. Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function from one magnetogram of active region NOAA 0061 (*lower right*). Other notations are the same as in Figure 4. Note that for this active region, the function $\zeta(q)$ is a nearly straight line with the vanishing degree of multifractality $\Delta h = 0.058$.

M-class flares and three X-class flares. One of them was the famous “Bastille Day” flare of X5.7 class launched on July 14, 2000 at 10:04 UT. Active region 0061, on the contrary, produced only 6 flares of X-ray class lower than C3 during the passage across the solar disk.

It also is worthwhile to note here that the third active region analyzed in the present study, NOAA AR 0501, showed both intermediate (between AR 0061 and AR 9077) flaring productivity, and intermediate multifractality (compare Figures 4, 8, 9). Our previous study of multifractality for 8 active regions (Abramenko et al. 2002) showed a similar tendency: active regions of low flaring activity tend to possess a lower degree of multifractality than active regions of high flaring activity do (see Figures 4,5 in Abramenko et al. 2002). Both results seem to suggest that the multifractality of the photospheric magnetic field may be related to the flare

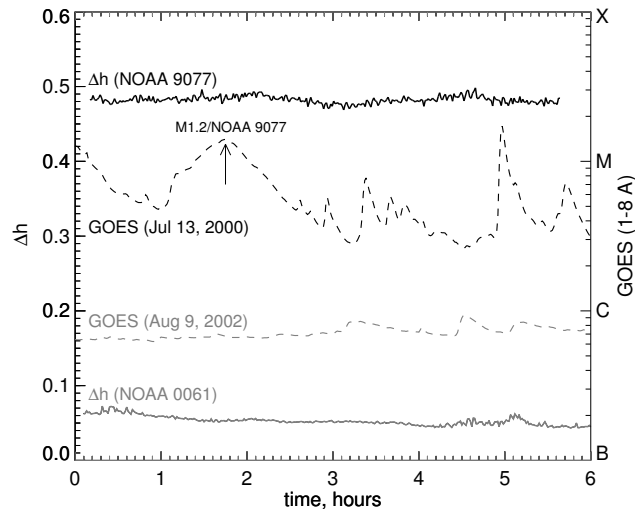


Figure 10. Time variations of the measure of multifractality, Δh (left axis), and GOES soft X-ray flux (right axis, dashed lines) plotted for six-hour time intervals for the two active regions. Data for NOAA AR 9077 (black lines) were obtained between 17:00 and 23:00 UT on July 13, 2000. The arrow marks the M1.2 flare occurred in NOAA AR 9077, whereas other peaks of the GOES flux are associated with another active regions on the solar disk. Data for NOAA AR 0061 (gray lines) refer to an interval between 11:00 and 17:00 UT on August 9, 2002.

productivity of an active region. Of course, a broad statistical study is needed to verify this suggestion.

Day-to-day changes in the flatness function for active region 0501 are presented in Figure 11. During the 24-hour time interval between magnetograms the flatness function became steeper ($\kappa = -0.53$ for the earlier data and -0.73 for the later data) and the range of multifractality, Δr , extended considerably, from $\Delta r = (5.5 - 19)$ Mm to $\Delta r = (2.5 - 22)$ Mm. The degree of multifractality also increased from 0.18 to 0.29. Comparison of the magnetograms, presented in the left part of Figure 11, suggests a reinforcement of a fragmentation of magnetic elements during the discussed time period. Indeed, the magnetic field in the Nov 19, 2003 magnetogram became more fragmented, especially inside

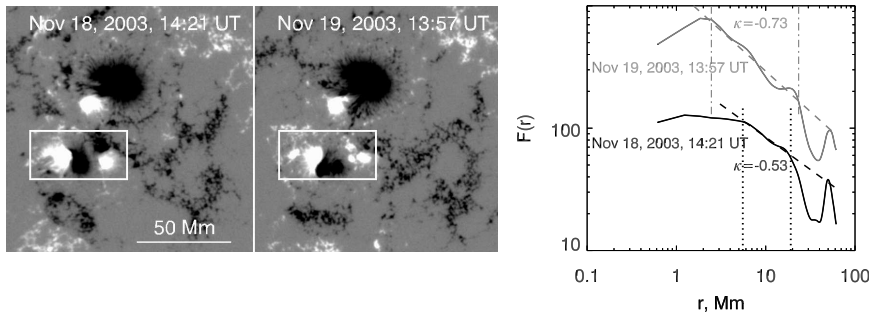


Figure 11. Two magnetograms of NOAA AR 0501 obtained on November 18, 2003 at 14:21 UT and on November 19, 2003 at 13:57 UT. An area where the fragmentation of magnetic elements is the most noticeable, is marked by white boxes. *Right:* - Flatness functions $F(r)$ for two magnetograms. Gray curve, corresponding to the later magnetogram, is shifted up into a “well-readable” position. The range of multifractality, Δr , for the earlier magnetogram is indicated by the dotted lines, whereas the more broad Δr for the later magnetogram is marked by dash-dotted lines. κ is the power index of $F(r)$ calculated as a linear best fit (*dashed lines*) inside the corresponding intervals Δr .

the white box, where some large magnetic structures disappeared while many small-scale features appeared in their places. Reinforcement of fragmentation seems to be the cause of the extension of the range of multifractality. In this case the increase of multifractality seems to be plausible, because the intensification of the fragmentation process may lead to a highly intermittent, multifractal structure of solar magnetic field (Abramenko & Longcope 2005).

5. Summary and Discussion

This paper presents further elaboration of the approach proposed by Abramenko et al. (2002) to analyze properties of multifractality of solar magnetograms. The approach is based on calculations of the structure functions $S_q(r)$ that are defined as the statistical moments of the increment of a field under study. Scaling characteristics of the structure functions allow to estimate the degree of multifractality, Δh , of the field. In the present study a new scaling characteristic of the structure functions, namely, the flatness function $F(r)$, is proposed for the analysis of multifractality.

It is shown that the scale interval of multifractality, Δr , can be visualized from the flatness function. Correct determination of Δr is important for estimation of the degree of multifractality and may also have independent physical applications in future. Determination of multifractality with the code proposed seems to be not dependent on parameters of the Gaussian background of the digitized image of the magnetic field.

Application of the code to magnetograms of solar active regions showed that the structure functions and their scaling characteristics differ for active regions studied. For a given active region, the shape of the functions may remain unchanged during several hours. However, the structure functions can noticeably change during a day. Thus, in NOAA AR 0501, the extension of the range of multifractality from $\Delta r = (5.5 - 19)$ Mm to $\Delta r = (2.5 - 22)$ Mm and the increase of the degree of multifractality from $\Delta h = 0.18$ to $\Delta h = 0.29$ was accompanied by a noticeable fragmentation of magnetic flux concentrations during a 24-hour period. This seems to be in a good agreement with the result reported by Abramenko & Longcope (2005), which suggests that one possible cause of the increase of multifractality in the photospheric magnetic field is the process of fragmentation of magnetic elements.

There are reasons to assume that multifractality has different character in flaring and flare-quiet active regions. Namely, flare-quiet active regions tend to display a lower degree of multifractality than flaring active regions do. Thus, in the present study, flare-quiet active region NOAA AR 0061 during 6-hours period of observations showed a very low level of multifractality $\Delta h = 0.060 \pm 0.007$, while in the flaring active region 9077 the degree of multifractality, determined during a 6-hour interval, was almost 10 times stronger, $\Delta h = 0.50 \pm 0.01$.

The amplification of multifractality means, on one hand, appearance of large fluctuations and, on the other hand, extension of areas of low fluctuations (see Figure 3). For the magnetic field, it is equivalent to emergence (or intensification) of small-scale flux tubes carrying very strong flux. However, emergence of large-scale (nearly homogeneous) magnetic structures may also contribute into the increase of multifractality. This increase may be an indication that a magnetic structure is driven to a critical state (Charbonneau et al. 2001, Vlahos & Georgoulis 2004) by gaining tangential discontinuities (Parker 1987, 1996) of various

length scales. In this case, the above suggestion about the possible relation between the character of multifractality and level of flare productivity seems to be reasonable and deserves further investigations.

The concept of multifractality of solar structures and, especially, solar magnetic fields can be used to gain deeper insight into the processes occurring in non-linear dissipative dynamical systems such as magnetized plasma in the solar atmosphere.

A byproduct of the proposed method is the possibility to automatically control images quality: the flatness function varies very gradually with time, so that any abrupt changes in the shape of flatness function are a signal of a possible damage of the image.

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