Multifractal Nature of Solar Phenomena

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Abstract Last decades, Solar Physics research demonstrated a considerable progress in understanding of macro-scale processes (e.g., the magnetohydrodynamic modeling of the heliosphere, modeling of large-scale coronal magnetic structures, etc.), on one hand and micro-scale phenomena (e.g., turbulence of solar plasma) on the other hand. Further progress seems to be associated with the realization of how various micro-scale processes are involved and manifested in the macro-scale behavior of the entire Sun. A similar problem unavoidably arises in studies of any, other than the Sun, non-linear dynamical dissipative system present in nature. Such systems, that can be placed in between a chaos and a completely determined structure, display a scale-invariant behavior. A broad class of models for scale-invariant structures is now elaborated based on the notion of a fractal, which can be mathematically described as an object with fractional dimensions. The goal of this chapter is to show how the concept of fractality and multifractality is currently elaborated in Solar Physics and how these ideas help us to understand the unpredictable behavior of our closest star. The first section provides introduction to multifractality and outlines physical connections between various aspects of this paradigm. Second section is devoted to reviewing the multifractality approach in the study of the Sun: we will discuss how high-order statistical moments, fractal geometry, multifractality, percolation theory, and the concept of the self-organized criticality are now applied to a broad range of solar phenomena: from explosive events of various scales to geometrical properties of observed filigree structures. The last section outlines new ideas and results based on the concept of multifractality.

1 What does the multifractality mean?

When starting to treat any physical system, one have to answer a fundamental question: whether the dissipation of energy can be neglected? In the case of positive answer, the system can be treated as a deterministic system that can be described by a set of differential equations and the uniqueness of the solution is fully determined by the initial conditions. However, for the majority of processes in nature, the energy dissipation cannot be neglected when one tries to obtain a realistic picture for their evolution. Such processes should be
treated as a nonlinear dissipative dynamical system (NDDS) and solar plasma is one of them. The universality of such processes (from forest fires and earthquakes to cosmic rays, magnetosphere sub-storms, solar flares, etc.) inspires our efforts to explore them.

The fact that the inner dissipation cannot be neglected anymore results in several important outcomes. First, small fluctuations in the initial state may result in significant deviations of the solution in the future that unavoidably drives the system to chaos. However, chaos can not be set in immediately, it takes some time to destroy a perfectly determined system. A specific state in the evolution of the system appears, when determinacy is lost, but a pure chaos is not reached yet, the state when the system acquires a fractal structure (recall the jagged coast line of Norway) as in spatial, so in temporal domains.

Nevertheless, complete chaos is unreachable due to nonlinear interactions between different scales (again in both spatial and temporal domains), that results in capability of a NDDS to rebuild large- and middle-scale substructures from small-scale chaotic fragments (recall the formation of large clusters in a turbulent interstellar medium, or the inverse magnetic helicity cascade in magnetohydrodynamics). This property of a NDDS results in the existence of an attractor in the phase space, i.e., an asymptotic limit of the solutions, which is not necessarily determined by initial conditions (see, e.g., Berge et al. 1988; Henighan 1997).

Second, a specific state between determinacy and chaos is, in turn, characterized by existence of self-invariance, when parts of an object somehow resemble the whole. To put it in more rigorous way, a self-similar, scale-invariant structure without any characteristic length appears, a fractal (Mandelbrot 1982; Feder 1988; Takayasu 1989; Schroeder 2000). A classical example of self-similarity is shown in Figure 1: a whole fern leaf resembles its part (framed), which, in turn, can be considered as a whole leaf and similar smaller part can be cut out, and so on. When we cover a fractal by a mesh, the number of boxes, \( N \), occupied by the fractal structure will be proportional to the scale length (the length of a sample box, \( r \)) in a fractional power, \( D_f: N(r) \propto r^{D_f} \). \( D_f \) is called fractal dimension and it is an essential characteristic of a fractal. Computer-generated fractals are self-similar over an infinite range of scales, and their geometrical properties can be adequately described by only one parameter, fractal dimension. However, in natural fractals self-similarity does not hold in the strict sense but does hold statistically over a finite range of scales. As soon as we deal with natural fractals, we immediately reveal that the slope of \( \log(N) \) plotted vs \( \log(1/r) \) may change inside different scale ranges (Figure 2). Physical reasons for this variable slope are the presence of nonlinear interactions between different scales (see, e.g. Berge et al. 1988; Biskamp 1993) and inhomogeneous character of dissipation (Monin and Yaglom 1975; Frisch 1995). Now the entire range of scales can be divided into an infinite number of subsets and for each subset its own self-similarity law holds with its own fractal dimension. Thus, a natural fractal may be considered as a multifractal, i.e., a superposition of an infinite number of monofractals. Local (in sense of scale) character of self-similarity of natural fractals is one of the reasons to make a step from monofractals to multifractals.

Another advantage that multifractals present is the possibility to take into account the distribution of density in a fractal. In the case of solar magnetic fields, we can obtain a frac-
tal when we define a magnetic area as a binary mask (Figure 3) equal to unity on the set of all pixels where the absolute value of the vertical component of the magnetic field, $B_z$, exceeds a given threshold (for example, 50 G, Figure 3, upper left frame), and to zero elsewhere. By using the box-counting method, for example, or some other methods (Takayasu 1989), we can calculate the fractal dimension of the binary mask, which characterizes complexity of its boundary. However, the fractal dimension can tell us nothing about the distribution of the magnetic field density within the mask. It is well known that the density distribution may not be homogeneous. Moreover, the density distribution largely determines the complexity of the magnetic structure and, therefore, the underlying physics. Thus, we have to incorporate the information on density. This can be done by introducing an hierarchy of masks defined by increasing threshold. The idea is illustrated in Figure 3: by specifying progressively higher thresholds, say, 500 G (lower left frame), 1000 G (upper right frame), we obtain new fractals and their parameters may vary. By analyzing these new monofractals, we can obtain addition of information on the distribution of increasingly dense patterns in the magnetic structure. The superposition of the infinite number of these monofractals results in a multifractal: a real distribution of magnetic fields on the solar surface (see lower right frame in Figure 3. Please, note that for simplicity sake, the sign of $B_z$ on the masks was disregarded.) To characterize the structure of a multifractal, a continuous spectrum of fractal parameters - a multifractal spectrum - is needed. The broader the multifractal spectrum, the more complex structure it represents.

Structures that arise in the course of evolution of dynamic systems, where dissipative processes may not be neglected, display highly intermittent character. Intermittency implies a tendency of a physical entity to concentrate into small-scale features of high intensity surrounded by extended areas of less intense fluctuations (Monin & Yaglom 1975; Biskamp 1993; Frisch 1995). Intermittency manifests itself via burst-like behavior in temporal and spatial domains. Figure 4 demonstrates the phenomenological difference between a Gaussian process (which is non-intermittent) and an intermittent process. Large fluctuations in the intermittent process are not as rare as in the Gaussian process, and they contribute significantly into the statistical moments, which leads to multifractality (see Frisch 1995 for more details). Thus, by analyzing multifractality of a structure, we also explore its intermittency. Generally speaking, intermittency and multifractality are two different terms for the same phenomenon. Historically, the term Intermittency is usually applied to time series analysis, whereas the term multifractality is used for spatial objects (Takayasu 1989; Frisch 1995). It is worth noting that when a field is intermittent, the structure of its energy dissipation can not be homogeneous but it is intermittent, too (Monin & Yaglom 1975; Frisch 1995). Thus, by exploring intermittency of solar magnetic fields, for example, one may also obtain valuable information on the behavior of magnetic energy dissipation (Lawrence et al. 1995; Abramenko 2003; Abramenko et al. 2003; Vlahos & Georgoulis 2004).

One of the traits of a multifractal structure (or intermittent process) is a non-Gaussian distribution function. As it follows from the central limit theorem, a Gaussian random process is caused by a sum of many independent random variables. The distribution function in this case is completely determined by two non-random parameters: the first and second
statistical moments (the mean value and the standard deviation) of the random variable. For an intermittent/multifractal process, the distribution function decreases (as the scale goes to infinity) slower than it does for a Gaussian process, which is caused by the increased role of large fluctuations in intermittent process (they are now much stronger and appear more frequently). The growth of the strong peaks population may occur for different reasons, however the most plausible one is that now the fluctuations are not a sum of independent (and approximately equal) random variables, but they rather are a product of many variables (as it happens in any fragmentation process or random multiplicative cascade). For an intermittent/multifractal variable, the distribution function is no longer determined by the first and second statistical moments. High-order moments play critical role and contribute significantly into the shape of the heavy tail of the distribution function.

To summarize, multifractality is a property of a dynamical system in which energy dissipation cannot be neglected. Multifractality is equivalent to intermittency and manifests itself as a highly jagged, porous structure in spatial domain and via burst-like behavior in time, when extended areas (intervals) of low fluctuations are intermittent with small areas of extremely large fluctuations.

There are several approaches to study multifractality. It can be diagnosed by i) analyzing of high-order statistical moments (e.g., calculation of distribution, structure, and flatness functions); ii) calculating fractal dimensions and multifractality spectra; iii) applying percolation theory; and iv) self-organized critically (SOC) theory. The list is far from being complete and it only includes the methods that are most frequently used in solar physics.

It is worth noting that ideas of multifractality and intermittency are applied also throughout a broad range of scientific studies, from interstellar medium (e.g., Falgarone et al. 1995; Cho et al. 2003; Kritsuk & Norman 2004; Balsara & Kim 2005) to biological and social systems (see numerous examples in, e.g., Schroeder 2000).

2 Studies of the Sun based on the multifractality ideas

2.1 Exploring high statistical moments

2.1.1 Distribution functions

Determination of the distribution function (or probability distribution function, PDF) from observations of a random process in question is usually the first and essential step of any study. Usually, a PDF is determined with one goal in mind: to find the mean value of the data set and its standard deviation. However, the PDF possesses much more information about the process. Strictly speaking, high-order statistical moments can be also evaluated from the PDF (Feder 1988; Takayasu 1989). When the PDF displays a heavy non-Gaussian tail, this is a manifestation of a highly intermittent and multifractal process, prone to eruption-like energy release. Moreover, this also indicates that the system, as a whole, is undergoing a non-linear dissipation with the energy interchange between different scales. Thus, a shape of the PDF is a powerful diagnostics tool. Figure 5 demonstrates
a qualitative difference between the distribution functions for a Gaussian and intermittent processes.

Distribution functions for a variety of solar processes and structures were derived as of now. First suggestion on a non-Gaussian behavior of our Sun appeared after pioneer space observations of the solar emission in X-rays.

**a. Power-laws reign in statistics of X-ray bursts.**

First solar X-ray experiments on Orbiting Solar Observatories, OSO-1 (White, 1964), OSO-5 (Frost, 1969), OSO-7 (Daltowe et al. 1974), Explorer 33 and Explorer 35 (Drake, 1971), provided extensive information on hard and soft X-ray bursts. First statistical study of various bursts parameters (Drake, 1971) revealed that distributions of the peak flux of bursts, their time-integrated flux, rise time, and decay time display the frequency distributions that are far from Gaussian. Observed distributions possessed a slowly diminishing at infinity tail and allowed a good fitting with an exponential function and/or a power law function. Later, many authors confirmed this results on the basis of subsequent space missions and provided a variety of power indices for the total energy release, peak count rate, and event duration. Most of them are reviewed by Crosby et al. (1993), Aschwanden et al. (1998) and Charbonneau et al. (2001).

**b. Distribution of the magnetic flux content in photospheric magnetic elements.**

Bogdan and co-authors (Bogdan et al. 1988) reported on heavy-tailed distribution of umbral areas of solar sunspots that stimulated other efforts to calculate and analyze similar distributions for elements of the magnetic field in the photosphere. Modern observational technique allows us to calculate the distribution function of the magnetic flux content in elements of the magnetic field at the photospheric level. In quiet Sun areas, where the magnetic elements are widely separated, PDFs can be determined relatively more easier than in an active region area, where elements are tightly packed to form clusters of various size and shapes. As a result, studies of the distribution of magnetic flux in quiet sun regions are more frequent. Wang et al. (1995) studied the dynamics and statistics of the quiet-sun magnetic fields using Big Bear Solar Observatory (BBSO) videomagnetograph data. They found the distribution function to be a power law with the power index of \(-1.68\) in areas with the magnetic flux in the range of \((0.2 - 1) \times 10^{18}\) Mx (intranetwork fields) and \(-1.27\) for flux in the \((2 - 10) \times 10^{18}\) Mx range (network elements).

Schrijver et al. (1997a) used high resolution data for a quiet sun area from the Michelson Doppler Imager (MDI, Scherrer et al. 1995) on-board of the Solar and Heliospheric Observatory (SOHO). They reported that the flux distribution follows an exponential law with a slope of approximately \(1 \times 10^{-18}\) Mx\(^{-1}\) in those areas where the flux ranges from 1 to \(5 \times 10^{18}\) Mx. The distribution of magnetic flux in flux concentrations in a plage area (outside of large sunspots) is less steep than that for the quiet sun, with the exponential slope slightly varying as a function of the flux Schrijver et al. (1997b). To explain the shape of the observed distribution, these authors modeled the distribution of flux assuming that three primary processes underlie the distribution of the magnetic flux: merging, cancellation and fragmentation of flux concentrations. Under this assumption, the exponential slope of a model distribution should vary inversely with the square root of the average flux.
density, and thus a satisfactory agreement between the observed and modeled distributions was reached in the range \((20 - 150) \times 10^{18}\) Mx.

Later, Abramenko and Longcope (2005) analyzed the distribution of flux content in magnetic elements in the range of flux \(\Phi > 10^{19}\) Mx for two well developed active regions. A particular attention was payed to the analytical approximation of the observed distribution, which were calculated from SOHO/MDI high-resolution line-of-sight magnetograms. The first active region NOAA 9077 observed on July 14, 2000 displayed very high flaring activity: it produced the famous Bastille day flare of X5.7 X-ray class, and about 130 lesser flares occurred during the active region passage across the solar disk. A magnetogram of this active region is shown in Figure 3 (lower right frame) and in Figure 13. The second active region NOAA 0061, observed on Aug 9, 2002, was a very quiet one: it produced several small C-class flares only. During the time interval under consideration, this active region produced no flares at all. One of its magnetogram is shown in Figure 14. During the MDI observations, both active region were located near the center of the solar disk, therefore, the measured line-of-sight component of the magnetic field very closely represents the \(B_z\) component of the magnetic field, which is normal to the solar surface. For each active region, we used a sequence of 248 magnetograms.

To calculate the PDF, we need first to locate and separate each magnetic element in a magnetogram. Our image segmentation code works as follows: we assumed that a given pixel belongs to a magnetic flux concentration when the absolute value of the magnetic field in it exceeds the specified threshold, i.e. \(|B_z| > p\). Calculations were performed for a set of thresholds: 25, 50, 75 and 100 G. The local peak of an individual concentration was determined as a pixel where both, \(d|B_z|/dx\) and \(d|B_z|/dy\), change their sign from positive to negative (here \(x\) and \(y\) are the current coordinate on a magnetogram). The next step was to outline the flux concentrations and to calculate their unsigned flux content. In the case when the unsigned flux density in a given pixel exceeds the threshold, the code determines to which peak a given pixel belongs by moving in a direction of the maximum gradient on the map of \(|B_z|\). When a pixel labeled as a peak is encountered, the flux and the area of the pixel are summed to the total flux and total area of a given flux concentration. The result of the application of this image segmentation code to an arbitrary \(40 \times 40\) pixels fragment of a magnetogram is shown in Figure 6.

In each magnetogram, approximately 600-800 concentrations were normally localized, depending on the threshold value. For the great majority of the concentrations in both active regions, the absolute value of the magnetic flux did not exceed \(8 \times 10^{20}\) Mx. This value was accepted as the upper limit of the flux range for the histogram. After normalizing each histogram for the total number of concentrations in a magnetogram, they were averaged over the total number of magnetograms, so that an averaged probability distribution function, PDF(\(\Phi\)), was obtained. These averaged PDFs for both active regions obtained with the threshold \(p = 50\) G are shown in Figure 7 (left panel).

To apply an analytical approximation to the observed distributions, we first determined the lower value of the magnetic flux, \(\Phi_{\text{cut}}\), below which the measured flux may be significantly affected by influence of noise, threshold, resolution, etc. Thus, we chose \(\Phi_{\text{cut}} = 10^{19}\)
Mx as flux cut-off above which we believe our sample is reasonably complete.

For an arbitrary magnetogram of each active region, we calculated the cumulative distribution function (CDF) for flux values $\Phi > \Phi_{cut}$. This function, $CDF(\Phi)$, giving the fraction of all concentrations with flux greater than $\Phi$, does not require the bin size be defined. The CDF(\Phi) for an arbitrary magnetogram of active region 9077 is shown in Figure 7 (right panel, red line).

To quantitatively analyze the obtained distributions, we attempted to fit the observed PDF with three different model distributions: log-normal, exponential and power law.

In a log-normal distribution (Aitchison & Brown 1957; Romeo et al. 2003) the logarithm of the magnetic flux is normally distributed. The corresponding PDF can be written as

$$PDF(\Phi) = \frac{1}{\Phi s(2\pi)^{1/2}} \exp \left[ - \frac{1}{2} \left( \frac{\ln(\Phi) - m}{s} \right)^2 \right],$$

for $\Phi > 0$. The function’s two parameters $m$ and $s$ are the mean and standard deviation of $\ln(\Phi)$. The expectation value, $\mu$, and the variance, $\sigma^2$, of log-normally distributed flux are defined in terms of these parameters

$$\mu = \exp \left( \frac{2m + s^2}{2} \right),$$

$$\sigma^2 = \exp(2m + 2s^2) - \exp(2m + s^2).$$

The mean flux, $\mu$, exceeds the median flux, $e^m$, by a factor $e^{s^2/2}$, which can be far greater than unity for large values of $s$. In the opposite limit, $s \ll 1$, the log-normal distribution approaches a Gaussian (normal) distribution and the mean approaches the median.

The averaged PDF was fit with expression (??) by performing a standard, unweighted least-squares Gaussian fit to the function $\Phi \cdot PDF(\Phi)$ versus $\ln(\Phi)$. The mean and standard deviation of this fit yield the parameters $m$ and $s$. Values for each active region for different threshold values are listed in Table 1. A typical example of the log-normal approximation in the range $\Phi > \Phi_{cut}$ is shown in Figure 7.

The second possibility, an exponential distribution, corresponds to the PDF

$$PDF(\Phi) = C_1 \exp(-\beta\Phi),$$

where $C_1 = \beta e^{\beta \Phi_{cut}}$ when using the range $\Phi > \Phi_{cut}$. We calculated $\beta$ as the negative slope when $\ln(PDF)$ is fit, by unweighted least-squares, to a linear function of $\Phi$. Our data for AR NOAA 9077 gave the value $\beta \approx 0.036 \times 10^{-18} \text{ Mx}^{-1}$ when $\ln(\Phi)$ was fit over a narrow flux range $(20 - 110) \times 10^{18}$. This agrees well with the value found by Schrijver et al. (1997b) for plage area inside an active region over the same flux range (see Figure 2 in Schrijver et al. 1997b). The result of the exponential approximation is shown in Figure 7 (right panel, blue curve).

The final possibility, a power law distribution, is given by the PDF

$$PDF(\Phi) = C_2 \Phi^{-\alpha}.$$
In an unweighted least-squares fit of $\ln(\text{PDF})$ to a linear function of $\ln(\Phi)$ the slope gives $-\alpha$. An example of the power law fit for the CDF of AR 9077 is shown in Figure 7 (right panel, green curve).

The quality of each fit is quantified using the Kolmogorov-Smirnov (K-S) test (Press et al. 1986). The observed CDF (shown by the thick red line in Figure 7, right) was computed from a single arbitrary magnetogram of NOAA AR 9077. The model CDF (violet, blue or green curve in Figure 7, right) uses parameters found by fitting the average PDF of the same active region. The K-S statistic $d$, the maximum absolute deviation between the model and the observed CDF, quantifies the likelihood that the data was drawn from the model distribution (Press et al. 1986). The “significance” is the probability of obtaining a greater value of $d$ if data were randomly drawn from the model distribution. For example, when the significance is smaller than 0.01, we may rule out the proposed model distribution with 99% confidence. The K-S statistic and significance are listed in Table 1 for the three distributions. The K-S test eliminates, with very high confidence, all distributions except the log-normal. The log-normal distribution is consistent with the CDFs from both active regions at every threshold value. The results above suggest that the magnetic flux in photospheric concentrations is log-normally distributed, which implies involvement of fragmentation process into evolution of magnetic fields.

In the solar photosphere and convective zone, two essential processes determine the interaction between turbulent plasma flows and magnetic flux tubes. On one hand, a magnetic element tends to be advected by the turbulent diffusion. On the other hand, turbulent motions tend to sweep the field lines together at convergence/sink points of the flow. Fragmentation and concentration processes are in a dynamic equilibrium in homogeneous stationary turbulence. Petrovay and Moreno-Insertis (1997) showed that in inhomogeneous and/or non-stationary situations, turbulent diffusion dominates the concentration causing turbulent erosion of magnetic flux tubes. The idea of the gradual disintegration of sunspots due to erosion of penumbral boundaries was first advanced by Simon & Leighton (1964). Later, Bogdan et al. (1988), on the basis of the observed log-normal distributions of umbral areas, argued that fragmentation of the magnetic elements may be an essential process in the formation of observed magnetic structures. The log-normal distribution of the flux content in magnetic flux elements of an active region suggests that the process of fragmentation dominates the process of flux concentration.

Assuming that the log-normality of flux concentration results from repeated, random fragmentations, we may attribute meaning to the distribution parameter. The variance, $s^2$, of $\ln(\Phi)$ is proportional to the number of independent fragmentations produced a given concentration from a single initial concentration (Monin & Yaglom 1975, Ch. 25). If the basic fragmentation process is similar in all active regions then $s^2$ is proportional to the time over which those fragmentations have occurred. Since the magnitude of $s^2$ for NOAA AR 9077 exceeds that for NOAA AR 0061, by a factor of 2.3, we may further infer that NOAA AR 9077 may be older than NOAA AR 0061 by approximately that factor. Alternatively, NOAA AR 9077 may have undergone more vigorous fragmentation over a comparable lifetime. The latter possibility may also explain the very different level of the
flaring activity in those active regions. Note, that a very intensive fragmentation of sunspots during several days before the Bastille day flare in NOAA AR 9077 were reported by Liu & Zhang (2001).

c. Distribution of speeds of coronal mass ejections as measured from LASCO coronographs

Another attempt to apply the distribution function technique to extract the underlying physics was performed by Yurchyshyn and colleagues (Yurchyshyn et al. 2005) in their analysis of initial speeds of coronal mass ejections (CMEs). CMEs are frequently associated with strong flares on the Sun and expand outward from the Sun, often toward Earth (and beyond) causing strong geomagnetic storms (see, e.g., reviews by Gopalswamy et al. (2006) and Schwenn et al. (2006)).

Yurchyshyn and colleagues studied the distribution of plane of sky speeds determined for 4315 CMEs detected by Large Angle and Spectrometric Coronograph Experiment on board Solar and Heliospheric Observatory (SOHO/LASCO). They found that the speed distributions for accelerating and decelerating events are nearly identical and to a good approximation they can be fitted with a single log-normal distribution. This study clearly demonstrates an heuristic idea: when an original distribution function (Figure 8) is very skewed and has a heavy tail, there is a good chance that the logarithm of the value in question will be distributed normally (see Figure 9).

The fact that the two groups of CMEs (accelerating and decelerating events) can be modeled by a single distribution may suggest that the same driving mechanism is acting in both slow and fast dynamical types of CMEs. What then can we learn from the fact that CME speeds are distributed log-normally? It is accepted that magnetic reconnection between independent magnetic systems plays a major role in the origin of a coronal ejecta. The log-normal distribution of the speeds of CMEs implies then the presence of nonlinear interactions and multiplicative processes in a system of many magnetic flux loops. When complexity of an evolving magnetic field reaches its peak so the magnetic configuration is no longer stable, a major instability as well as a entire spectrum of smaller scale instabilities may develop in the system of many loops and fluxes, regardless of the assumed initial configuration (Maia et al., 2003). Thus, Kaufmann et al. (2003) interpreted sub-millimeter pulse bursts at the CME onset as signatures of energy release due to small-scale instabilities in the magnetic field. In this sense, the reconnection process in a CME model can be viewed as a multiple reconnection process or as an avalanche of multiple reconnection sites (Lu & Hamilton 1991; Vlahos et al. 1998; Wang et al. 2000; Charbonneau et al. 2001; Abramenko et al. 2003; Kaufmann et al. 2003) in a fragmented magnetic field (Kuklin 1980; Bogdan 1988; Petrovay et al. 1999, Abramenko & Longcope 2005). However, to what extent the multiple reconnection process of fragmented solar magnetic fields is responsible for the log-normal distribution of the CME speeds is yet to be determined.

d. Distribution functions for parameters of the magnetic field in the solar wind

We will discuss the solar wind in section 3 of this chapter. Here we review only a couple of studies that concern the PDFs of the solar wind data.

Burlaga & Lazarus (2000) reported a log-normal distributions of the speed, density and
proton temperature measured at the solar wind streams near Earth. Authors emphasized an intermittent, spiky character of measured fluctuations. Recently Vasquez and coauthors (Vasquez et al. 2007) analyzed distributions functions of some parameters of the solar wind observed with ACE spacecraft near Earth. Namely, they determined discontinuities in the structure of the magnetic field that characterize dynamics of processes in solar wind’s streams on their way from the Sun to the interplanetary space. In particular, the observed distribution function for the separation time intervals between two consecutive discontinuities is shown in Figure 10. The log-normal analytical fit was found to be the best best fit for the observed function. For other analyzed parameters, namely, the width of discontinuity, the spread angle, the increment of the magnetic field density at the discontinuity, log-normal fits were found as best analytical approximations. Analysis of observed distribution functions and comparison with distribution functions derived from numerical experiments led authors to conclude that the Alfvenic turbulence is the most plausible mechanism of the origin of magnetic discontinuities in the solar wind plasma. In particular, the log-normal distribution of separations between discontinuities may indicate that a multiplicative random cascade produces the discontinuities.

2.1.2 Scaling of Structure Functions

Solar plasma, beginning from the base of the corona (the photosphere) up to the outer heliosphere, is in a state of fully developed turbulence. Any system in such a state is an excellent example of a non-linear dynamical dissipative system. Many properties of such systems hold for a turbulent media, as well. At the same time, a turbulent system can be a subject for specific tools elaborated in the framework of the turbulence theory, and those tools eventually provide us with information on their multifractal/intermittent properties. So that multifractality of a turbulent system can be explored by using tools of the turbulence theory. The structure function analysis is one of such tools.

Since Kolmogorov’s study (Kolmogorov 1941, hereafter K41), many models have been proposed to describe the statistical behavior of fully developed turbulence. In these studies, the flow is modeled using statistically averaged quantities such as velocities, and structure functions play significant roles. Structure functions were first introduced by Kolmogorov and they are defined as statistical moments of the increments of a turbulent field $u(x)$ as

$$S_q(r) = \langle |u(x+r) - u(x)|^q \rangle,$$  

(6)

where $r$ is a separation vector, and $q$ is a real number. Structure functions, calculated within the inertial range of scales, $r$, ($\eta \leq r \leq L$, where $\eta$ is a spatial scale where the influence of viscosity becomes significant and $L$ is a scaling factor for the whole system) are described by the power law (K41; Monin and Yaglom 1975; Frisch 1995):

$$S_q(r) \sim (\varepsilon_r(x) \cdot r)^{q/3} \sim (r)^{\zeta(q)}.$$  

(7)

where $\varepsilon_r(x)$ is the energy dissipation, averaged over a sphere of size $r$. 

The function $\zeta(q)$ describes one of the most important characteristics of a turbulent field. In order to estimate this function, Kolmogorov assumed, that for fully developed turbulence (turbulence at high Reynolds number, i.e., when the inertial force highly exceeds the viscous force), the probability distribution laws of velocity increments depend only on the first moment (mean value), $\bar{\varepsilon}$, of the function $\varepsilon_r(x)$. Replacing $\varepsilon_r(x)$ in equation (2) by $\bar{\varepsilon}$ we have

$$S_q(r) \sim (\bar{\varepsilon} \cdot r)^{q/3} = C \cdot r^{q/3}, \quad (8)$$

where $C$ is a constant. As a result, function $\zeta(q)$ is defined as a straight line with a slope of $1/3$:

$$\zeta(q) = q/3. \quad (9)$$

Kolmogorov further realized (see also formulation of Landau’s objection concerning the original K41 theory in Frisch 1995) that such an assumption is very rigid and turbulent state is not homogeneous across spatial scales. There is a greater spatial concentration of turbulent activity at smaller scales than at larger scales. This indicates that the energy flow and dissipation do not occur everywhere, and that the energy dissipation field should be highly inhomogeneous (intermittent) and also follows a power law,

$$\langle (\varepsilon_r(x)^3/3) \rangle \sim \bar{\varepsilon}.$$

Equation (??) is now referred to as the refined Kolmogorov’s theory (hereafter, refined K41) of fully developed turbulence (Kolmogorov 1962a; Kolmogorov 1962b; Monin and Yaglom 1975; Frisch 1995).

One can see from equation (??) that the function $\zeta(q)$ deviates from the straight $q/3$ line. It has been suggested that the deviation is caused by the scaling properties of a field of energy dissipation.

Equation (??) for velocity increments has been carefully checked many times over the last four decades and experimental results thus confirmed this suggestion (see Gurvich et al. 1963; Anselmet et al. 1984; Schmitt et al. 1994; Frisch 1995 and references in). In Figure 11 we reproduce a result published in Schmitt et al. (1994). One sees that $\zeta(q)$ is an increasing concave outward function crossing the K41 line at $q = 3$, where $\zeta(3) = 1$, which satisfies both K41 and refined K41 theories as long as $\langle (\varepsilon_r(x)^3/3) \rangle = \bar{\varepsilon}$.

Important information on a turbulent field can be derived from structure functions $\zeta(q)$, which can be, in principle, obtained from experimental data. For example, the value of the function at $q = 6$ deserves special attention because it defines a power index

$$\beta \equiv 1 - \zeta(6) \quad (13)$$
of a spectrum \( E^{(e)}(k) \) of energy dissipation \( \varepsilon(x) \) that is defined as

\[
E^{(e)}(k) \sim k^\beta,
\]

where \( k \) is a wave number. Observed values \( \beta \) for a velocity field at fully developed turbulence were found to be in the range of \(-0.8 \leq \beta \leq -0.6\) (Gurvich et al. 1963; Anselmet et al. 1984).

Having \( \zeta(q) \), as derived from experimental data, and by using equation (14) one can calculate the scaling exponent \( \tau(q/3) \) in equation (17) for the energy dissipation field.

The derivative of \( \zeta(q) \),

\[
h(q) = \frac{d\zeta(q)}{dq},
\]

can also be obtained by using the \( \zeta(q) \) function (Figure 11, right frame). The deviation of \( h(q) \) from a constant value is a direct manifestation of intermittency in a turbulence field, which is equivalent to the term multifractality in fractal terminology.

According to fractal theory, an intermittent structure consists of subsets, each of which follows its own scaling law. In other words, all values of \( h \), within some range, are permitted and for each value of \( h \) there is a fractal set with an \( h \)-dependent dimension \( D(h) \) near which scaling holds with exponent \( h \). The values of \( h \) and \( D(h) \) are related by the Legendre transform (Frisch 1995).

Thus, multifractality of a field is manifested via concavity of \( \zeta(q) \) function. We accepted the parameter \( \Delta h \equiv h_{\text{max}} - h_{\text{min}} \) to be an estimation of the degree of multifractality of the field (Abramenko 2003). Indeed, larger values of \( \Delta h \) are associated with a broader set of monofractals that together form the resulting multifractal. A typical example of the structure functions for an active region’s magnetogram is presented in Figure 12.

The weakest point in the above technique is the determination of the scale range, \( \Delta r \), where the slope \( \zeta(q) \) is to be calculated. To visualize the range of multifractality, \( \Delta r \), we suggest to use the flatness function (Abramenko 2005b), which is determined as the ratio of the fourth statistical moment to the square of the second statistical moment. Frisch (1995) suggested to use higher statistical moments to calculate the (hyper-)flatness, namely, the ratio of the sixth moment to the cube of the second:

\[
F(r) = S_6(r)/(S_2(r))^3.
\]

We used Equation (17) to calculate \( F(r) \) and, for simplicity, we will refer to \( F(r) \) as the flatness function. For monofractal structures, the flatness is not dependent on the scale, \( r \). On the contrary, for a multifractal structure, the flatness grows as a power-law, when the scale \( r \) decreases. Usually, for natural fractals, the range, \( \Delta r \), of the flatness’s growth is bounded (see lower left frame in Figure 12). Possible causes of the boundness of the interval of multifractality were not comprehensively studied yet. The slope of flatness function, \( \kappa \), determined within \( \Delta r \), is related to the degree of multifractality (Figure 12).

a. Structure functions of the magnetic field in active regions

We applied the structure function method to the real magnetograms of different solar active regions (Abramenko et al. 2002; Abramenko 2005b). The result of calculations for two
active regions, NOAA 9077 and 0061, are presented in Figures 13 and 14, respectively. We see that scaling behavior of the structure functions is rather different in these active regions. In active region NOAA 9077 there exists a well-defined range of scales, \( \Delta r = (4 - 23) \) Mm where flatness \( F(r) \) grows with the power index \( \kappa = -1.17 \) as \( r \) decreases. Function \( \zeta(q) \) is concave and the corresponding \( \Delta h \approx 0.5 \). This implies a multifractal structure of the magnetic field in this active region. To the contrary, in NOAA AR 0061 (Figure 14) the flatness function undulates around the horizontal line, which implies a monofractal character of the magnetic field. The function \( \zeta(q) \) is nearly a straight line with a vanishing value of \( \Delta h \approx 0.05 \).

Time profiles of \( \Delta h \) for the two active regions are compared in Figure 15. Two six-hour data sets allowed us to conclude that the non-flaring NOAA AR 0061 persistently displays lower degree of multifractality, as well as lower X-ray flux, than the flaring NOAA AR 9077 does.

It also is worthwhile to note that the third active region NOAA AR 0501 that was analyzed with the structure function method (Abramenko 2005b), showed both intermediate flaring productivity, and intermediate multifractality as compared to AR 0061 and AR 9077 (compare Figures 12, 13, 14). Other examples of the relationship between multifractality and flaring productivity are shown in Figure 16 from Abramenko et al. 2002. One may suggest that higher degree of multifractality of the magnetic field may be associated with stronger flare productivity of an active region. Of course, a broad statistical study is needed to verify this suggestion.

Day-to-day changes in the flatness function for active region NOAA 0501 are presented in Figure 17. During the 24-hour time interval between magnetograms, the flatness function became steeper (\( \kappa = -0.53 \) for the earlier moment and \( -0.73 \) for the later moment) and the range of multifractality, \( \Delta r \), extended considerably, especially, at the small-scale side. The degree of multifractality also increased from 0.18 to 0.29. These changes seem to be related to the intensification of the fragmentation process, which, in turn, may lead to a highly intermittent multifractal structure of solar magnetic field (Abramenko & Longcope 2005). Comparison of the magnetograms, presented in the left part of Figure 17, confirms the conclusion on the reinforcement of fragmentation during the discussed time period. Magnetic fields in the 13:57 UT magnetogram are more fragmented, especially inside the white box, where several large magnetic structures disappeared, while many small-scale features appeared in their places. Reinforcement of the fragmentation seems to be the cause of the extension of the range of multifractality toward smaller scales due to involvement into fragmentation of progressively smaller scales.

b. Structure functions of the of the coronal EUV emission images

Time series of full-disk images of the solar corona obtained by the Extreme Ultraviolet Imaging telescope (EIT, Delaboudiniere et al 1995) on board the SOHO spacecraft in the spectral line Fe XII 195 Å were recently analyzed by Uritsky and co-authors (Uritsky et al. 2007). Two approaches for the spatio-temporal analysis of the coronal brightness above a background EUV threshold were used. Results of the treatment of the coronal brightness structures in the framework of the SOC theory will be discussed below, in Section 2.4 of
this chapter. Here we will discuss the results on intermittency of the solar corona explored by the structure functions technique.

EUV emission structures observed near the center of the solar disk (Figure 18, lower panel) were treated as a continuous turbulent field and the corresponding structure functions were calculated for each instant of time with Equation (??). A typical example of such structure functions is shown in Figure 19.

To avoid non-linearity in the structure functions, the extended self similarity (ESS) method was applied under the assumption that \( \zeta(3) = 1 \) (Kolmogorov scaling). The main idea of the ESS method (Benzi et al. 1993) is to plot \( \log(S_q(r)) \) versus \( \log(S_3(r)) \) and to determine the relative scaling exponent, \( \alpha_3(q) = \zeta(q)/\zeta(3) \). Thus, inside the linear range of \( S_3(r) \), we have \( \alpha_3(q) = \zeta(q) \). One important advantage appears here: the range of linearity in the \( \log(S_3(r)) - \log(S_q(r)) \) plot is much more extended than that in the \( \log(r) - \log(S_q(r)) \) plot (compare the insert in Figure 19 with the original \( S_q(r) \)). Therefore, the relative scaling exponent, \( \alpha_3(q) \), can be determined with accuracy that is much higher than that for the function \( \zeta(q) \). A disadvantage is the unavoidable assumption \( \zeta(3) = 1 \). All of the above can be repeated for \( \zeta(4) = 1 \) (Kraichnan scaling).

The relative scaling exponent derived from observational data show the well-pronounced deviation from non-intermittent Kolmogorov-type scaling (Figure 20, dotted straight line), which manifests the highly intermittent nature of the hot solar corona.

c. Structure functions of the photospheric velocity fields

In 1999 Consolini and co-authors (Consolini et al. 1999) reported first attempt to apply the structure functions approach to diagnose the intermittency in the photospheric line-of-sight velocities. High-resolution Dopplergrams obtained in two spectral lines (Fe I 557.61 nm and Cl 538.03 nm) with the THEMIS telescope were analyzed by applying the ESS technique. The authors concluded the presence of intermittency in the photosphere. Their results showed that plasma at deeper layer (level of Cl 538.03 nm line formation) is more intermittent than that at higher layer (data from the Fe I 557.61 nm line). This finding allowed them to speculate that intermittency is relevant to the photospheric convection.

2.2 Fractal dimensions and multifractality spectrum

For the first time, a fractal dimension appears in the context of solar magnetic fields in the pioneering publication by Tarbell et al. (1990). The authors studied high resolution (0.3 to 0.5 arcsec) magnetograms of an active region recorded with the Swedish Solar Observatory on La Palma, Canary Islands. They concluded: “A threshold magnetogram resembles a fractal set, similar to those in Mandelbrot (1982). We can estimate the fractal dimension of this set from a plot of area versus length scale and find self-similar behavior with a dimension of about 1.6 over the entire range considered, from 0.16 to 10 arcsec.” This statement was the first experimental proof of fractal nature of magnetic fields on the solar surface, however heuristic ideas about fractality of solar magnetic fields were in air long before, see, for example, Zeldovich et al. (1987).

Later on, more extensive analysis of these excellent data was performed by Schrijver et al. (1992) and Balke et al. (1993). This team analyzed an area on the magnetogram, which
included a plage region on the periphery of an active region NOAA AR 5168 so that the fields under study were of moderate intensity. In above studies, the influence of magnetograms noise was studied, as well as a correction of a magnetogram with the modulation transfer function was applied. Authors concluded that "the difference in fractal dimension between the original magnetogram ($D_f = 1.76$) and the enhanced [MTF-corrected- V.A.] magnetogram ($D_f = 1.54$) can be understood as caused by the higher resolution in the enhanced image: with increasing resolution, more details can be observed in clusters, and the fractal dimension will differ more strongly from the Euclidian dimension $D_f = 2$" (Balke et al., 1993). We are inclined to think that such a difference, at least, partially, is caused by the multifractality of the magnetic structure: transition to higher resolution unavoidably involves smaller scales, where $D_f$ is different (see Figure 2). Indeed, the slope in the plot of $\log(\text{Area})$ versus $\log(\text{Scale})$ [see Figure 3 in Balke et al. (1993) and Figure 1 in Schrijver et al. (1992)] is not constant over the entire range of scales. Thus, even these first studies of fractal dimension in solar magnetograms give a hint that the observed filigree structure is not a monofractal, but rather a multifractal.

Later, Tao et al. (1995) undertook an attempt to numerically simulate spatial distribution of the surface magnetic field. Their simulation was based on the numerical solution of the induction equation with the diffusivity item. This very simple model showed a set of key properties in the behavior of magnetic elements: the crucial role of the transverse velocities in formation of a fractal-like magnetic structure, the loss of memory of the initial conditions during the long-time evolution, as well as the multifractal nature of the resulting flux distribution. During the subsequent decade, numerical simulations of multifractality and intermittency of the solar structures were performed by other groups (e.g., Cadavid et al. 1994; Einaudi et al. 1996; Georgoulis et al. 1998; Dmitruk et al. 1998; Janssen et al. 2003; Nigro et al. 2004).

Fractal analysis was also applied to structures of the solar corona observed in extreme ultra violet (EUV) spectral range by Gallagher et al. (1998). Thresholded images of the corona observed in the range of temperature from $10^4$ to $10^6$ K above a quiet sun area displayed a fractal dimension in the range of $D_f = (1.3 - 1.7)$, which is in agreement with that reported by Tarbel et al. (1990) and Balke et al. (1993) for the photospheric magnetic structures. We are inclined to think that observed variability of the fractal dimensions $D_f$ is related, at least, partially, to the multifractality of the corona.

Lawrence and co-authors (Lawrence et al. 1993) presented a detailed review [see also Lawrence et al. 1996] of the multifractal measures and, for the first time, applied the technique of multifractality for magnetograms of quiet and active regions on the Sun. Their results demonstrated multifractality of these both types of magnetic structures. Based on these observational results, Cadavid et al. (1994) suggested a multifractal model of small-scale solar magnetic fields.

Mathematical definitions and calculation formula for the multifractal spectrum can be find, for example, in Feder (1988), Takayasu (1989), Frisch (1995), Schroeder (2000).

Meunier (1999) studied time variations of the fractal dimension of the longitudinal magnetic field on time scales of about 20 months and reported no particular trend with time and
no significant correlation with Wolf numbers. Ireland et al. (2003) found the correspondence between the fractal dimension of the photospheric magnetic field and the Hale class of an active region.

Recently, a statistical study of the fractal dimension of the magnetic field in active regions was performed by McAteer et al. (2005). Nearly $10^4$ active region images acquired during 7.5 years of MDI observations were analyzed. Authors reviewed the existing methods to calculate the fractal dimension and gathered results of previous calculations of fractal dimension in active regions (see Table 1 in McAteer et al. 2005). They calculated the fractal dimension of the boundary of an active region: for a smooth boundary (low complexity), $D_f = 1$, while $D_f \rightarrow 2$ as the boundary becomes more and more serrated. They found that when $D_f > 1.2$, then M and X class flares are probable within the next 24 hours. Authors concluded that "solar flare productivity exhibits an increase in both the frequency and GOES X-ray magnitude of flares from regions with higher fractal dimension", i.e., with higher complexity of the active region’s boundary. Thus, even being treated as a monofractal, the structural organization of the magnetic field exhibits relationship with capability to produce explosive events/flares. This inference seems to be natural, as long as the multifractality in the spatial domain implies highly intermittent (burst-like) behavior in time.

Burst-like behavior in time, in turn, reminds of a percolation process, when a long-lasting period of quietness can abruptly be replaced by an avalanche covering the entire porous array - percolation cluster. This analogy suggests about an ultimate relationship between the multifractality/intermittency and the evolution of a percolation cluster toward an avalanche.

### 2.3 Percolation Theory Approach

A phenomenon of percolation, being a subject of very clear and simple description, brings us to a variety of fractal structures. We will illustrate the basic notion of the percolation theory with an example of a forest fire (Figure 21), i.e. percolation on a two-dimensional lattice.

Let us represent a forest as a square point lattice on which trees independently occupy nodal points with a probability, $p$, so that $p$ is equal to the ratio of the total number of trees to the total number of nodal points, $p = n_t/N$. Let the bottom row of trees be ignited (Figure 21 a). We will assume that a burning tree will always ignite all of the trees located at adjacent nodal points after one unit of time (i.e., there exists a bound between two adjacent nodes when both of them have a tree). If the density of trees, $p$, is very low, the fire will die out quickly and will not penetrate far from the ignition site.

What will happen when the density of trees increases? Trees, connected with bounds, form clusters of trees. Fire can propagate along bounds until it reaches the boundary of the cluster. On Figure 21 b, by igniting the bottom row, we onset five separated energy release events (or five avalanches of energy release) and the largest of them reaches only the 6-th row. When the entire available energy of all burning clusters is exhausted, the fire will die out with no damage for trees located above the 6-th row. In this case, the size of the fire and the amount of released energy are mostly determined by the largest avalanche. In the case
illustrated in Figure 21b, \( p = 168/(28 \cdot 20) = 0.30 \), and the size of the largest avalanche is \( \lambda = 5 \) pixels. Let us add, say, another 140 trees (Figure 21c). Now, \( p = 0.55 \) and \( \lambda = 17 \) pixels, however, the fire still did not reach the upper boundary of the lattice. In this situation, addition of any new tree can be critical and could lead to catastrophic results (assuming that the upper row is, say, a real estate boundary). So that when the trees density increases up to 0.59 (Figure 21d), fire will reach the upper boundary. With addition only the 22 trees to the existing 308 trees, a continuous chain of trees (the global, percolation cluster) is formed, which allows fire to penetrate quickly from the lower to upper boundary of the lattice. Thus, the size of fire can grow up to the size of the entire forest (lattice). This is a state of percolation.

First essential property of percolation is the abrupt transition from a state of local coherence to a state of global coherence, in other words, a threshold character of dissipation. Numerical modeling on large 2D lattices showed that a cluster, extending over the entire lattice, appears for the first time when the progressively increasing \( p \) reaches the value of \( p_c = 0.59257 \pm 0.0003 \) (Feder 1988). When the size of the lattice, \( L \), goes to infinity, the probability of appearance a global coherent cluster decreases to zero when \( p < p_c \) and it goes to 1 when \( p > p_c \). The abrupt transition from low to high probability occurs near the percolation threshold, \( p = p_c \). If we define the "mass" of a cluster, \( M(L) \), as the number of nodal points belonging to the cluster, then the mass of the largest cluster is related to the lattice size, \( L \), as

\[
M(L)_{L \to \infty} \sim \begin{cases} 
\ln L & \text{if } p < p_c, \\
L^D & \text{if } p = p_c, \\
L^E & \text{if } p > p_c.
\end{cases}
\] (17)

Therefore, at the percolation threshold, when \( p = p_c \), the largest coherent cluster, also called an inner percolation cluster, is a fractal with a fractal dimension \( D_f \).

The power-law relation between the scale and the mass of clusters with a fractional power index at the percolation threshold is the second essential property of percolation. At the bound percolation (from a nodal point to another one) on a 2D lattice, \( D_f = 91/48 \approx 1.895 \) (Feder 1988). The distance, at which the connectedness of nodal points inside a cluster still holds, is determined by a connectedness length, or correlation length, that is usually denoted as \( \zeta \), or \( \lambda \). Phenomenologically, \( \zeta \) corresponds to the averaged size of clusters on the lattice, or to the correlation length (in turbulence terminology). In our example of the forest fire, it corresponds to the size of the fire. The explicit expression for \( \zeta \), which involves the so-called gyration radius of clusters and the probability distribution function for the size of clusters, can be found, for example, in (Feder 1988). An important diagnostic value of the correlation length is that it increases as the system evolves toward the critical state of percolation.

The probability of percolation, \( P_N(p) \), defined as \( M(L)/L^2 \), averaged over a large number of realizations, \( N \), at the percolation threshold is determined as \( P_N(p) \sim (p - p_c)^{\beta} \) when \( p > p_c \) and \( p \to p_c \). The index \( \beta \) is equal to \( 5/36 \approx 0.14 \) for the 2D percolation and \( \approx 0.4 \) for the 3D percolation (Feder 1988). In the later 3D case, the percolation corresponds very well to the phenomenon of magnetic phase transition, when the local arrangement of
magnetic moments increases with the temperature decreasing down to the critical value, $T_c$, below which the magnetic moments are arranged similarly over the entire sample and we have got a magnet.

Interpretation of a solar flare phenomenon as a phase transition caused by abrupt percolation of electric currents in a turbulent current sheet was suggested by Pustilnik (1997, 1998). Analyzing plasma instabilities of a turbulent current sheet hosting a flare, author concluded that the equilibrium state of the current sheet is extremely unstable due to various plasma instabilities present: tearing modes, pinch-type instabilities, overheating of turbulent regions in the current sheet, splitting of the current sheet at conductivity discontinuities. This results in an extremely inhomogeneous final state of the flare current sheet. Thus, the current sheet can be treated as a random resistors network consisting of "bad resistors" - turbulent low conductivity domains and "good resistors" - normal plasma domains. Propagation of the electric current through such medium is a percolation process with the fundamental properties: threshold regime of dissipation that explains the explosive nature of flares, and universal power-law distributions of statistical parameters of flares.

Pustilnik has also showed that the percolation in the turbulent current sheet is accompanied by the processes that are, in our opinion, physical realization of the so-called redistribution rules in the SOC theory. These processes are: i) redistribution of currents in the current sheet due to permanent stochastic rebuilding of the resistors network and the current conservation law; ii) redistribution of the heat conductive flux from the heated turbulent domains into surrounding cool plasma which leads to change of the threshold current. This circumstance provides a bridge between the percolation theory and the SOC theory and partially explains why a bulk of interest was shifted towards the SOC theory during the last decade. The latter offers much more broad possibilities for numerical modeling of avalanche processes (that are ultimately the percolation processes) in application to a solar flare phenomenon (see the next section of this chapter).

Successful attempts were undertaken to apply the percolation theory to explain the distribution of magnetic patches over the surface of the Sun. Thus, Wentzel & Seiden (1992, see also Seiden & Wentzel, 1996) modeled appearance (emergence under buoyancy) of sunspots on the solar surface as the percolation through the convective zone. The idea to exploit the percolation approach was stimulated by observations that newly emerging active regions prefer to be located inside the boundary of existing sunspot regions (Zirin 1988, Harvey 1993, Harvey & Zwaan 1993). Under a certain choice of model parameters (such as "the probability that the rise of one flux tube stimulates rise of a neighboring flux tube" and a parameter related to the diffusion of flux tubes), Seiden and Wentzel succeeded to obtain synthetic synoptic maps of "sunspots", which reproduced the intermittent character of the real maps: persistent vast empty regions, corresponding to coronal holes and quiet-sun areas, and compact areas of enhanced magnetism - active regions. Moreover, the size distribution of simulated active regions was in a good agreement with observations reported by Harvey & Zwaan (1993).

Analysis of real solar magnetograms in the framework of the percolation theory was presented in the two publications discussed above (see previous section of this chapter),
namely, in Schrijver et al. (1992) and Balke et al. (1993). A binary mask (similar to that shown in Figure 3) was calculated with a threshold of 500 G, which produced an ensemble of clusters formed by nearest neighbor pixels with the magnetic flux density exceeding the threshold. Area, $A$, of each cluster was determined and compared with the size $L$ of the smallest box which can contain the cluster [see Figure 3 in Balke et al. (1993) and Figure 1 in Schrijver et al. (1992)].

The correlation length, $\zeta \approx 3$ arcsec, was calculated by using the gyro-radius technique (Balke et al. 1993). The slope of the $\log A$ versus $\log L$ plot inside the spatial range $L < \zeta$ allowed to calculate the fractal dimension $D_f$, that was found to be lower than the fractal dimension $D_f \approx 1.89$ of a cluster at the state of percolation, $p = p_c$. Authors simulated an artificial percolation clusters by randomly filling a 2D lattice of the same size as the magnetogram. The density of filling, $p = (\text{number of filled pixels})/(\text{total number of pixels})$, varied from 0.49 to 0.55. A conclusion was made that "the observed distribution corresponds best with the simulations for which $0.52 \leq p < p_c = 0.59275". These studies have showed that the magnetic field structure, at least, inside plage areas and below scale of the 3 arcsec, can be treated as a randomly distributed magnetic flux forming a percolation cluster under the percolation threshold. This inference became in future a basis for elaboration of the random walk theory to explore processes of magnetic flux diffusion in the solar photosphere (e.g., Lawrence and Schrijver 1993). "This broad [percolation] theory offers a consistent explanation of geometric properties of both the magnetic flux distribution in instantaneous patterns and of the observed displacements of magnetic flux, without any gouge constants or fudge factors remaining unexplained" (Schrijver et al. 1992).

### 2.4 Self-Organized Criticality (SOC) approach

Behavior of highly intermittent/multifractal dynamical systems can be also modeled with the SOC theory. The connection between the multifractality and SOC theory is not as obvious, as it is between the multifractality and the percolation theory. Nevertheless, fractal nature of dissipative non-linear dynamical systems is the basis for the concept of self-organized criticality. Indeed, as we saw in the previous section, an assembly of clusters at the state of percolation possesses a property of fractality, moreover, they form a multifractal system. On the other hand, a process of percolation is an abrupt transition into another state that for many systems implies an explosive event, which, in turn, is an avalanche in the SOC theory.

The SOC theory is mainly focused on analysis of physical conditions and processes that can drive the system to the critical state of catastrophes. In this sense, SOC approach might be more useful for understanding the underlying physics of catastrophes, as compared to the pure geometrical analysis offered by the percolation theory. This also may explain why the SOC approach in solar physics applications is mostly concentrated on statistics of solar flaring processes attracting a strong attention of researchers (Lu & Hamilton 1991; Lu at al. 1993; Vlahos et al. 1995; MacKinnon et al. 1996; Georgoulis et al. 2001; Krasnoselskikh et al. 2002; Podladchikova et al. 2002; Podlazov & Osokin 2002; see also a review by Charbonneau et al. 2001 and references in).
It seems to be nearly impossible to avoid the traditional explanation of the SOC ideas by using the sand pile example. Let us consider falling grains of sand on a table surface, so that a pale of sand gradually forms. At the beginning, the height of the pile is increasing and the slopes are steepening. Rare, small-scale avalanches may occur. When the angle of the slope exceeds some critical magnitude \((\theta > \theta_c)\), adding one single grain to the pile can provoke a large sand slide (avalanche) that may extend from the top to the very bottom of the pile. The avalanche will redistribute the steepness of the slopes which may result in occurrence of the similar critical situation \((\theta > \theta_c)\) at another place, which will cause another avalanche. Continuous falling of grains recurs the system to the previous situation, and so on. The system is now in a state of dynamical equilibrium, when the continuous energy input (slow driving) is balanced by abrupt dissipation events. The state is called a critical state. It is important to emphasize that the system reaches such state in a natural way, without any tune factors involved. Because of that, the critical state is called the self-organized criticality. Statistical parameters of avalanches obey the power law and the geometry of avalanches displays properties of multifractality (see, for example, Schroeder 2000, chapter 17; McIntosh & Charbonneau 2001).

An excellent review of the SOC theory applications in solar physics was presented by Charbonneau et al. (2001). Here we restrict our self to a brief outline of the problem. Generally, a routine consists of two steps: i) the SOC modeling of a time set of avalanches/flares, and ii) comparison of the statistics of simulated events to that obtained from real observations.

Usually, the modeling starts with specifying of a (uniform) magnetic field on a 2D lattice. After the injection of a randomly placed perturbation, a certain parameter (often, the field gradient or electric current, depending on a model) of the field is recalculated over the lattice. If, at some nodal point, the parameter exceeds a specified threshold, then the point is classified as a critical and is attributed to an avalanche. The excess of the parameter at the critical point is then redistributed over adjacent nodal points (the redistribution rule is frequently a delicate issue and it differs from one model to another). After the redistribution, the excess of the threshold may occur at any of the adjacent nodes so that this node is also classified as critical and belonging to the same avalanche. This second nodal point also undergoes the redistribution, and so on, until the avalanche is exhausted. After that, next random disturbance is injected at a random location and the entire routine is repeated. At the very beginning of the model evolution, the total magnetic energy of the system, \(\sum B^2 / 8\pi\), increases. However, after some time period, a dynamical equilibrium is reached and the critical state is settled. Now, the total energy, on average, is constant being superposed by fluctuations due to episodic avalanches. The energy released in avalanches is a small fraction of the total energy of the system. Strictly speaking, an exact prediction of the location, time and size of the oncoming avalanche is impossible.

The above description is a simplified presentation of the general idea for such kind of modeling. Real routines can be much more complicated (some delicate methodological aspects of the SOC concept and related techniques are discussed in Buchlin et al. 2005). A common point of all proposed SOC methods is a presentation of differential equations on a
discrete lattice. Recently, a new approach to use continuous presentation of equations was suggested by Belanger et al. (2007). Authors argue that this new method may allow some possibilities for short-time forecasting of avalanches, which is extremely important in solar flares application.

Usually, the SOC modeling produces statistical distributions of simulated avalanches that are similar to that obtained from solar observations (see references in Charbonneau et al. 2001). The reason for such a good correspondence might be in the satisfactory accomplishment of the requirements of the SOC theory for the solar atmosphere. According to Parker’s conjecture (Parker 1983a, 1983b, 1988), slow driving of the system is performed by random motions of footpoints of magnetic flux tubes caused by convection in the photosphere and below. Various disturbances in the magnetic field propagate up into the corona, which creates braided and intertwined magnetic flux tubes (or, in other words, numerous tangential discontinuities), thus driving the coronal magnetic field to the SOC state. When the density of discontinuities exceeds some critical level, an avalanche of fast small-scale reconnection events (a flare) occurs. Thus, the requirement of slow driving and sudden energy release is met. Furthermore, the amount of energy released in a typical flare is a small fraction of the total magnetic energy stored in a typical active region’s magnetic configuration. The medium hosting the flare is an extremely complex, multiscale system of coronal magnetic fields that possesses fractal properties, for which a plausible analog for the redistribution rule can be suggested (for example, redistribution of currents in a resistor network by Pustilnik 1997, see previous section of this chapter). Recently, new ideas were suggested to explain a chain reaction of avalanche-like reconnections under solar conditions, for example, a mechanism of the secondary instability proposed by Dahlburg et al. (2005) and a chain reconnections in flaring process proposed by Kusano et al. (2004).

The ultimate connection between the SOC paradigm and the concept of intermittency for the hot solar corona was demonstrated in a paper by Uritsky et al. (2007) that was discussed above (Section 2.1.2b of this chapter). The shape of the structure functions’ scaling exponents (Figure 20) undoubtedly demonstrates the intermittent character of the coronal brightness structures (the 3D structure in Figure 18). For the same continuous brightness structures, authors also calculated thresholded snapshots (top panel in Figure 18) that outline the most heated places in the corona the spectral line can detect. Having the time series, it is possible to determine the lifetime of each cluster of energy release. These clusters were treated as avalanches of energy release. The most important advantage here is that the avalanches were not modeled but rather derived from solar observations. The statistics of avalanches were calculated separately for periods of minimum and maximum of the 23-th solar cycle. The probability distribution functions for the total emission flux, $E$, the peak area, $A$, and the lifetime of avalanches, $T$, are reproduced in Figures 28 - 24, respectively. The distributions were successively fitted with the power law functions which is a necessary condition for the SOC concept. Although the authors’ inferences are based on coronal ultraviolet emission data, we are inclined to think that the conclusions are applicable for a broad class of solar phenomena as well. This is because the SOC theory captures an essential property of the solar plasma: multifractality in spatial domain and
Intermittency in time.

3 Intermittency in the heliosphere

This is a very broad topic, that involves a voluminous body of literature and a long history. The whole notion of fractality came into solar physics from heliospheric studies, see, for example, pioneering papers by Burlaga & Klein (1986) and Burlaga (1991). Also a great variety of mathematical methods to analyze multifractal/intermittent systems were adopted by solar physicists from numerous studies of the solar wind. We can recommend a recent review by Bruno et al. (2005) and references in.

Even though the solar wind is now observed mostly near Earth, it is an essential part of the Sun’s evolution. Solar wind streams are high-speed flows of charged particles propagating in the magnetized medium under the accelerating forces, which largely exceed the gravitational force of the Sun. A possible mechanism of the acceleration is still under debates (see, e.g., Fisk & Gloeckler 2006), however its ultimate energy source is believed to be on account of the Sun.

The solar wind mainly consists of two components: slow solar wind (speed \( \sim 400 \) km/s), which originates from areas of closed magnetic fields on the Sun, and fast solar wind (speed \( \sim (600 - 800) \) km/s), which originates from areas of open magnetic fields on the Sun, co-called coronal holes. Predominant location of coronal holes varies with the solar cycle: during a solar minimum, coronal holes (of opposite polarity) are concentrated at the poles, whereas during the solar maximum, numerous coronal holes appear at low latitudes, near the solar equator and therefore, Earth becomes a direct target for fast solar wind streams emanated from low-latitude coronal holes.

Because the state of fully developed turbulence is reigning in the heliosphere, various methods of the turbulence theory are frequently used to study intermittency (e.g., Burlaga & Ness 1998; Bruno et al. 2005; Zimbardo et al. 2006; Kaghashvili et al. 2006). Along with this, solar wind is extensively studied with various traditional fractal techniques, such as i) analysis of fractal geometry (Polygiannakis & Moussas 1994; Milovanov & Zelenyi 1999); ii) structure functions (Carbone et al. 2004; Bigazzi et al. 2006); iii) the shell model (Boffetta et al. 1999; Lepreti et al. 2004); iv) the phase coherence method to study non-linear interactions (Koga et al. 2006); v) distribution function technique (Bruno et al. 2001; Lepreti et al. 2004; Leubner & Vörös 2005; Vasquez et al. 2007, the later was discussed in Section 2.1.1d of this chapter). This list is far from being complete and the only aim it pursues is to show a large effort devoted to this kind of research during last decade.

4 Conclusion

In this chapter, we attempted to demonstrate that a broad class of processes and systems on the Sun are neither completely deterministic, nor totally chaotic. They rather are in an intermediate state, which unavoidably possesses a property of multifractality in spatial
domain and intermittency in time. We schematically summarize our inferences concerning the place and the role of intermediate processes on diagrams in Figures 25 and 26.

How this knowledge can help us in our efforts to understand the nature in general and the Sun, in particular?

First of all, let us consider a methodological aspect. By applying a purely deterministic or purely chaotic approach for any new phenomenon, we may soon realize that the obtained result gives us only one-sided view while some important aspects might escape from our consideration. Besides, processes of integration and differentiation over a fractal measure drastically differs from those undertaken over a Lebesgue (continuous) measure. The same can be said about the averaging over a fractal measure: our traditional tools (such as the central limit theorem, Gaussian statistics, the least mean square technique) might be inapplicable.

As to the usefulness of the multifractality notion for various physical aspects of understanding of solar phenomena, it is worth to mention the long-standing problem of appearance of low plasma conductivity in the corona, especially during a flare. To explain a solar eruption on the scale of an active region that can last of about 100 minutes, it is necessary to imply the presence of super strong electric currents \((\sim 10^{10} A/km^2)\) inside a very thin layers \((< 100 m, \text{Priest 1982})\). A fractal concept of coronal magnetic fields can easy meet these requirements. Indeed, a self-similar fractal allows existence of super thin branches (magnetic sheets or tubes) whereas a percolation state implies formation of super strong currents at singular branches of the cluster.

Another intriguing aspect of solar flaring is prediction of flares. A voluminous body of literature (to mention a few, Falconer et al. 2003; Wheatland 2004; Abramenko 2005a; Song et al. 2006; Qahwaji and Colak, 2007; for more references, see Leka and Barnes 2007) and meetings is devoted to the problem, however no dramatic success is achieved as yet. We would like to mention here a conclusion that Leka and Barnes (2007) made from a very thorough and extensive set of four publications devoted to the flare forecast problem: “Our result suggest that the state of photospheric magnetic field at any single time has limited bearing on the occurrence of solar flares”. When keeping in mind a fractal nature of the phenomenon, is becomes natural that it is impossible to exactly predict a flare. We can only say whether or not an emerging active region reached the critical state and is able to produce a flare, or it is still in a state of accumulating the energy, when the probability of strong flares is very low. Realization that only a probabilistic statement of the question is physically reasonable, could help much in a treatment of the flare forecasting issue.

Very often studies produce different fittings to the observed distribution functions when analyzing the same solar phenomenon. Which approximation is true? Exponential, power law, or log-normal? What scaling exponents are closer to reality? The multifractal concept tells us that there is no unique answer for this question. The answer depends on the range of scales selected for the fitting: the slope of the distribution may vary for different ranges of scales. For example, for a narrow range, observational data can be successfully approximated by a linear fit in a double-logarithmic plot (a power law), however on a broader range, the observed distribution may be curved. A log-normal fit is capable to capture the
curvature of observed PDFs on a broad range of scales, being therefore more successful than the power law fit. Meanwhile, one should keep in mind that all above fittings are only models aimed to represent a multifractal system with a finite number of scalar parameters. Which is impossible in principle. An adequate description requires, at least, a continuous spectrum of parameters.

A widely discussed problem of the heating of the solar corona to multi-million degree temperatures (see, e.g., a recent review by Klimchuk, 2006) strongly relies on the highly inhomogeneous character of coronal magnetic fields and on ultimate interactions of different spatial scales, which both are attributes of multifractality. For example, the so-called direct current (DC) mechanism explains coronal heating as a result of numerous small-scale magnetic reconnections (nanoflares) that occur in places of anomalous resistivity of coronal plasma at numerous magnetic field discontinuities, which, in turn, are generated by random photospheric motions of magnetic flux tubes. This scenario perfectly satisfies both percolation and SOC concepts. Moreover, during last years a progress has been made in explaining why individual large-scale coronal loops do not expend (in their crossection) with height as they should do in force-free field approximation. Klimchuk et al. (2006, 2007) argued that a coronal loop, when treated as "bundles of unresolved, impulsively heated strands" can posses some very specific properties that will differ a real loop from a modeled. This "suggests that the coronal magnetic field is comprised of elemental flux strands that are tangled by turbulent convection. These strands are so small that many tens of them are contained within a single TRACE loop. We suggest that this fine structure is a critical missing ingredient of the extrapolation models and that a combination of footpoint shuffling and coronal reconnection can explain the observed loop symmetry. This has important implications for coronal heating."

Thus, ideas of multifractality, intermittency, the percolation theory, and the SOC theory, all of them share deep common roots, which is caused by natural evolution of non-linear dynamical dissipative systems. Our Sun is one of such systems, and solar physics can gain much by utilizing these concepts, along with a rich set of developed tools, in further understanding of our closest star.

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References


Figure 1: An example of a self-similar structure. A framed part of the fern leaf is similar to the whole leaf.

Figure 2: A slope of $\log(N)$ versus $\log(1/r)$ defines the fractal dimension, $D_f$. For natural fractals (i.e. for multifractals), the slope can change for different scale ranges.
Figure 3: An hierarchy of binary masks of a solar magnetic structure ($B_z$, lower right frame). Each binary mask (left frames and upper right frame) is equal to unity (white) on the set of all pixels where the absolute value of the magnetic field exceeds a given threshold (noted above the frames) and zero (black) elsewhere. Superposition of infinite number of such masks/monofractals results in the observed magnetic structure, which is a multifractal. For simplicity sake, the sign of $B_z$ is not considered.
Figure 4: Illustration of phenomenological difference between a Gaussian process (upper frame), which is a non-intermittent process of extremely rare large fluctuations, and an intermittent process (lower frame), which is a multifractal due to the contribution of large fluctuations (which are not so rare now) into the statistical moments. Horizontal axis may represent a time or a spatial coordinate.
Figure 5: Qualitative comparison of different distribution functions. When plotted in linear scales (upper left frame), it is rather difficult to diagnose the analytical shape of distributions: exponential, power law and log-normal functions look similar, all of them essentially exceed the Gaussian curve at large scales and pass below it at small scales. In the linear-logarithmic plot (upper right frame), the exponential function [Eq. (??)] is a straight line, whereas the power law function [Eq. (??)] is a straight line in the double-logarithmic plot (lower left frame). The log-normal function [lower right frame, Eq. (??)] is slightly curved in the double-logarithmic scales. Thus, the appearance of the observed distribution in different presentations may give a clue for an analytical approximation.
Figure 6: A map of the absolute value of the magnetic field, $B_z$, for an arbitrary $40 \times 40$ pixel area of the AR9077, which illustrates the processing of the image segmentation code. The image is scaled in the range of values $0-1200$ G. The white diamonds mark the position of the peaks of flux concentrations. The white lines outline the boundaries between the flux concentrations. Total unsigned flux inside each closed boundary was calculated.

Figure 7: Left - Averaged probability distribution functions (PDFs) of the magnetic flux content in flux concentrations for NOAA AR 9077 (red line) and for NOAA AR 0061 (green line). The best log-normal fits in the range of flux $\Phi > 10^{19}$ Mx to the averaged PDFs are shown by violet line for NOAA AR 9077 and by dark green line for NOAA AR 0061. Right - Observed cumulative distribution function (CDF, the thick red line) calculated in the flux range $\Phi > 10^{19}$ Mx from the magnetogram of NOAA AR 9077 obtained at 08:12 UT on July 14, 2000. The thin violet curve marks the log-normal function, $LN(m, s)$, with parameters $m = 2.20$ and $s = 1.49$, as they were obtained from the averaged PDF. The thin blue curve denotes the the best exponential fit with parameters $C_1 = 0.0135$ and $\beta = 0.0572$, calculated as a best fit to the averaged PDF. The thin green curve marks the power law function with $C_2 = 0.275$ and $\beta = -1.45$, calculated as a best fit to the averaged PDF. All three approximations are calculated over the flux range $\Phi > 10^{19}$ Mx. The level of significance of the Kolmogorov-Smirnov test, “sig”, for each type of approximation, is noted. The threshold is 50 G.
Figure 8: Distribution of the observed speeds of 4315 CMEs (vertical bars). The width of the bar is 70 km s$^{-1}$. The solid line represents a single log-normal fit to the observed data, while the dashed line is a sum of a Gaussian and a log-normal fits. Courtesy of V. Yurchyshyn.

Figure 9: Distribution of the logarithm of velocity, $\ln(v)$, determined for all 4315 CMEs (vertical bars). The solid line represents normal fit to the observed data. Courtesy of V. Yurchyshyn.
Figure 10: Histogram of the magnetic field discontinuity number as a function of separation in minutes between each discontinuity and the one which follows from January 10 to 13, 2001. The long tail of the distribution is evident. The thin curve denotes the log-normal fit to the data points.

Figure 11: Parameters of fully developed turbulence calculated by using structure functions defined from measurements of wind velocities in the earth’s atmosphere. Left - exponents $\zeta(q)$ of structure functions are shown as a function of order $q$ (experimental data from Schmitt et al., 1994); right - plot of the derivative $h(q)$ of function $\zeta(q)$ calculated by using (??).
Figure 12: Structure functions $S_q(r)$ (upper left) calculated from a magnetogram of active region NOAA 0501 by Equation (??). Lower left: flatness function $F(r)$ calculated from the structure functions by Equation (??). Vertical dotted lines mark the interval of multifractality, $\Delta r$, where flatness grows as power law when $r$ decreases. The interval $\Delta r$ is also marked in upper left frame. $\kappa$ is the power index of $F(r)$ determined within $\Delta r$. The slope of $S_q(r)$, defined for each $q$ within $\Delta r$, is $\zeta(q)$ function (upper right), which is a concave for a multifractal and straight line for a monofractal. Lower right: function $h(q)$ is a derivative of $\zeta(q)$. The interval between the maximum and minimum values of $h(q)$ is defined as a degree of multifractality, $\Delta h$. 
Figure 13: Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function from one magnetogram of active region NOAA 9077 (lower right). $\kappa$ is a power law index of $F(r)$ calculated inside $\Delta r$. The degree of multifractality for this magnetogram $\Delta h = 0.48$. Other notations are the same as in Figure 12.
Figure 14: Structure functions $S_q(r)$, flatness function $F(r)$ and $\zeta(q)$ function from one magnetogram of active region NOAA 0061 (lower right). Other notations are the same as in Figure 12. Note that for this active region, the function $\zeta(q)$ is a nearly straight line with the vanishing degree of multifractality $\Delta h = 0.058$. 
Figure 15: Time variations of the measure of multifractality, $\Delta h$ (left axis), and GOES soft X-ray flux (right axis, dashed lines) plotted for six-hour time intervals for the two active regions. Data for NOAA AR 9077 (red lines) were obtained between 17:00 and 23:00 UT on July 13, 2000 and data for NOAA AR 0061 (green lines) refer to an interval between 11:00 and 17:00 UT on August 9, 2002.
Figure 16: Scaling exponents $\zeta(q)$ of structure functions of order $q$ calculated for eight active regions by Abramenko et al. (2002). The straight dotted line has a slope of $1/3$ and refers to the state of Kolmogorov turbulence. The NOAA number and the strongest flare (X-ray class/optical class) of each active region is shown. Increase of the flaring activity of active regions (from the top down to the bottom) is accompanied by general increase in concavity of $\zeta(q)$ functions.
Figure 17: Two magnetograms of NOAA AR 0501 obtained on November 18, 2003 at 14:21 UT and on November 19, 2003 at 13:57 UT. An area where the fragmentation of magnetic elements is the most noticeable, is marked by white boxes. Right: Flatness functions $F(r)$ for two magnetograms. Green curve, corresponding to the later magnetogram, is shifted up into a “well-readable” position. The ranges of multifractality, $\Delta r$, are marked by dotted lines. $\kappa$ is the power index of $F(r)$ calculated as a linear best fit (dashed lines) inside the corresponding intervals $\Delta r$. 
Figure 18: Two views of the coronal EUV structure above an active region on the Sun. 
*Upper panel:* a snapshot used to construct avalanches (brightness above a threshold \( w_a = 350 \)). *Lower panel:* a snapshot of the continuous brightness of the corona at the same time instant. Courtesy of V. Uritsky.
Figure 19: Structure functions of coronal brightness. Each function is normalized by its maximum value. Inset: ESS scaling of the structure functions, The horizontal bar indicated the interval used for calculation of the relative exponents, ζ(q)/ζ(3). Courtesy of V. Uritsky.
Figure 20: Relative scaling exponents obtained by ESS method. Deviation from the straight K41 line implies intermittency. At solar maximum, the corona is more intermittent than it is at solar minimum. Comparison with the She-Leveque (SL) model and the Müller-Biskamp (MB) model is shown (for details see Uritsky at al. 2007). Courtesy of V. Uritsky.
Figure 21: A forest fire illustration of the state of percolation. As the density of trees, $p$, increases (from $a$ through $d$), the correlation length $\zeta$ (a typical size of local fires/avalanches) increases, too. When $p = 0.55$ (frame $c$), addition of only few trees can result in triggering of a global avalanche, when the fire penetrates through the entire forest, so that the state of percolation is reached (frame $d$).
Figure 22: Probability distribution functions of the total emission flux, $E$, of avalanches shown in the top panel of Figure 17. The data obtained for four different thresholds $w_a = k < w >$ with $k = 0.4$ (squares), $k = 0.8$ (diamonds), $k = 1.2$ (triangles), and $k = 1.6$ (circles). The data points related to the solar minimum (min) are shifted downward for better reading. The calculated indices of the power law approximation, $\tau_E$, are noted. Courtesy of V. Uritsky.

Figure 23: Probability distribution functions of the peak area, $A$, of avalanches shown in the top panel of Figure 17. Notations are the same as in Figure 21. Courtesy of V. Uritsky.
Figure 24: Probability distribution functions of the lifetime, $T$, of avalanches shown in the top panel of Figure 17. Notations are the same as in Figure 21. Courtesy by Vadim Uritsky.
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<td>A set of equations includes a dissipation term; a solution (if any) is not determined by a boundary/initial conditions</td>
<td>the central limit theorem; Gaussian statistics; the least mean square technique</td>
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Figure 25: The chart to illustrate the intermediate location of multifractal systems
Figure 26: Continuation of the chart presented in Figure 25.
Table 1: Parameters of the log-normal fit and K-S statistic and significance for the three distributions

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