

Green Function method for Potential Field Calculations

V.I. Abramenko

Big Bear Solar Observatory, 40386 N. Shore Lane, Big Bear City, CA 92314, USA

Calculations of a potential magnetic field above an active region are based on the solution of the Neumann Boundary Value Problem (NBVP) for the Laplace equation.

Let us specify a Cartesian coordinate system x, y, z so that plane x, y is tangential to the solar surface and vector \vec{z} is normal to this plane. A vertical component B_z of a magnetic field \mathbf{B} is then specified inside an area S that lays in the plane (x, y) and the center of S coincides with the origin of the coordinate system.

Thus, for a potential field \mathbf{B} we have

$$\mathbf{B} = -grad \phi \quad (1)$$

$$div\mathbf{B} = 0, \quad (2)$$

where ϕ is a scalar potential. Substitution of \mathbf{B} in eq. (2) by eq. (1) gave us the Laplace equation for scalar potential: $\nabla^2\phi = 0$. By supplementing this equation with boundary conditions defined at S we obtained the Neumann boundary value problem for the Laplace equation:

$$\nabla^2\phi = 0 \quad (3)$$

$$\left. \frac{d\phi}{dz} \right|_S = B_z^{obs}. \quad (4)$$

There exist several approaches to solve this problem. We chose a method of direct integration of Green's function. Namely, we used the fact that the solution to the NBVP in a half-space $z \geq 0$ is determined as a potential of a single layer, provided that $B \rightarrow 0$ when $r \rightarrow 0$ faster than $1/r$, conditions which are reasonable in the case of solar magnetic fields. Thus,

$$\phi(x, y, z) = \frac{1}{2\pi} \int \frac{B_z(x_1, y_1)}{((x - x_1)^2 + (y - y_1)^2 + z^2)^{1/2}} dx_1 dy_1. \quad (5)$$

Here the integration is assumed to be done over an infinite plane at $z = 0$. In numerical calculations the integration is substituted by summing over a grid specified on the finite area S and this substitution is the main source of computational errors. In our code, we smoothed Green's function

$$G(x, y, z) = \frac{1}{((x - x_1)^2 + (y - y_1)^2 + z^2)^{1/2}} \quad (6)$$

over each cell of a grid and then analytically corrected the influence of sampling.

Numerical testing showed that the mean-square deviation of an analytical solution from the numerical one obtained with the routine did not exceed 2% and azimuthal differences were no larger than 3 grad when the net flux vanishes. The present routine is not sensitive to the magnitude of the net flux over a magnetogram: even when the imbalance of the magnetic flux is significant (30-40%) the simulated magnetic field may deviate from the exact solution near the boundaries of a magnetogram no more than by 10-15%. The method was originally published in Abramenko (1986).

Transverse components of the observed magnetogram are then compared to the horizontal components of the potential magnetic field and the field orientation is corrected based on the assumption that the observed transverse fields makes an acute angle with the potential horizontal field.

REFERENCES

Abramenko, V.I. 1986, Bull. of the Main Astronomical Observatory in Pulkovo "Solnechnye Dannye" , N. 8, 83